RAMPEN MANAGEMENT

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Optimizing ambulance reallocation during mass casualty incidents
A comparative analysis of the potential of location models in the Belgian context

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Executive summary

Topic

This thesis investigates the optimization of ambulance reallocation during mass casualty incidents (MCIs), with a particular focus on the Belgian Emergency Medical Services (EMS) system. Mass casualty events, ranging from natural disasters to terrorist attacks, require a rapid scaling of medical response, often overwhelming existing EMS capacity. Given the budgetary and operational constraints typical in semi-public healthcare systems like Belgium's, the most feasible strategy is to temporarily reallocate existing baseline EMS assets to meet surge demand.

However, Belgium currently lacks a unified, theoretical framework to guide this reallocation process. EMS assets (ambulances, Paramedical Intervention Teams [PITs], and Mobile Urgency Groups [MUGs]) are distributed based on historical practices and hospital network structures rather than predictive, data-driven models. This fragmented system leads to inefficiencies in both routine operations and emergency response. This thesis explores whether established facility-location models can provide a structured, efficient, and context-appropriate method for reallocating EMS assets during a crisis without compromising baseline coverage.

Method

The study adopts a two-step methodological framework:

A. Scoping Literature Review

A systematic search was conducted across PubMed and Google Scholar to identify theoretical models applicable to EMS planning. After screening over 3,000 records and applying rigorous filtering criteria including novelty, relevance to the Belgian context, methodological transparency, and feasibility, 13 high-potential models were selected. These include deterministic, probabilistic, and dynamic models, each offering different strengths depending on the context and data availability.

B. Scenario-Based Simulation

To assess real-world applicability, five crisis scenarios were designed, varying in geography (urban vs. rural), EMS demand levels, and MIP activation intensity. A simulation using 2024 EMS availability data measured each model's performance across multiple dimensions: coverage, equity, efficiency, and computability. These were benchmarked against two heuristic approaches commonly used in Belgian EMS dispatching but which have never been formally evaluated.

Results

Simulation outcomes demonstrate that optimization-based location models significantly outperform current heuristic methods, particularly in high-demand urban settings. Key findings include:

- **Coverage:** Models like MCLP and FLEET maximize population coverage within critical response times, even under resource-constrained conditions.
- **Equity:** The use of Gini coefficients revealed that probabilistic and hybrid models achieved more equitable distribution of EMS services, avoiding over-concentration in high-density areas at the expense of rural regions.
- **Efficiency**: Several models reduced total EMS travel distances and minimized average response times, thereby improving operational efficiency and asset utilization.
- **Computability**: Most deterministic and probabilistic models delivered results within acceptable computation times, confirming their suitability for real-time or near-real-time deployment. Dynamic models, although conceptually strong, were limited by Belgium's current technological constraints.

Conclusion

This thesis confirms that location theory offers valuable tools for improving EMS asset allocation during MCIs. In the Belgian context, where resource scarcity, decentralized governance, and legacy practices dominate EMS planning, the integration of theoretical models can enhance both surge response and baseline service continuity. Deterministic and probabilistic models, particularly MCLP, TEAM, and MEXCLP, demonstrated practical viability and outperformed existing heuristic methods in nearly all evaluation metrics.

While full implementation of dynamic models may require future infrastructural investments, the immediate adoption of transparent, well-established models could provide a cost-effective and impactful policy improvement. By bridging theoretical modeling with real-world operational data, this thesis contributes actionable insights for policymakers, EMS planners, and crisis response coordinators seeking to build a more resilient and responsive emergency healthcare system in Belgium.

Declaration of Artificial Intelligence usage

Artificial Intelligence (AI) tools were used in the preparation of this thesis in a manner consistent with the academic integrity policies of the University of Antwerp[1]. AI was never used to generate ideas, conduct analysis, or contribute original content. Its use was strictly limited to assisting with language refinement, such as grammar, sentence structure, and clarity.

To ensure that the AI did not introduce new content or context, the tool was provided with bullet-point summaries of pre-written sections. Prompts were explicitly worded to prohibit the generation of additional material beyond the given input. Furthermore, all AI-assisted output was carefully reviewed and edited by the author to verify that no content was altered or introduced erroneously and to guard against potential hallucinations or inaccuracies. The final text reflects the author's own reasoning, structure and conclusions.

Acknowledgements

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Acronyms

AED automated external defibrillators

EMS Emergency Medical Services

GIS geographical information system

MCI mass casualty incident

MeSH medical subject headings

MUG Mobile Urgency Group

PIT Paramedical Intervention Team

1 Introduction

1.1 Casualty transport during mass casualty incidents

Surge Emergency Medical Services, a subset of surge capacity, refers to the ability of an Emergency Medical Services (EMS) system to rapidly expand its day-to-day operations in order to adapt to a mass casualty incident (MCI) [2]. These MCIs are described as incidents (natural or man-made) that generate more patients in such a short time span that they can overwhelm locally available resources when using their routine procedures [3, 4]. During such a mass casualty event, the healthcare system must find a balance between the heightened demand for resources during a crisis response, the delivery of high-quality baseline EMS, all while opertaing within budgetary and capacity constraints. While baseline and surge Emergency Medical Services (EMS) differ in many aspects, from transferring a large number of patients to the emergency department, the increased amount of staff, resources and logistics required to treat them, this thesis will only address the aspect of transport logistics [5].

Developing surge capacity involves a significant commitment of money and time, especially since financial incentives have often eliminated all excess capacity in search of efficiency and reduced overhead expenses [6]. Therefore, the most common and cost-effective strategy to handle the logistical aspects of surge capacity is to reallocate existing EMS assets that normally serve baseline EMS. Some healthcare systems opt for a hybrid approach which includes having supplementary, often more specialized units on standby [7]. This strategy balances economic efficiency with emergency preparedness, but it creates a critical decision point: how to select a subset of EMS assets that should be reassigned during surge events?

When policymakers implement a healthcare system where surge capacity requires a significant use of baseline EMS assets, they have the obligation to reflect on measures they can implement to reduce the impact of the unavailability of these assets on everyday baseline EMS. Preferably a plan is prepared in advance to assist in the selection process of this subset of assets.

1.2 The Belgian Context and Research Gap

While the basic principles of surge capacity and resource reallocation work across different healthcare systems, their implementation is highly dependent on the context. National regulations, EMS infrastructure, funding mechanisms and operational culture will all influence how these principles need to be deployed [8, 9]. Belgian policymakers, like many of their counterparts across the European Union, have sought to control rising healthcare expenditures in their (semi-) publicly funded systems. In doing so, Belgium has chosen to limit the number of available EMS assets. However, unlike some other nations, it lacks a unified, theoretical framework to determine the appropriate quantity of resources and their optimal positioning.

Belgium's multi-tiered approach employs a three-tiered approach each with its own resource planning systems:

- **Ambulances** are typically added in an ad hoc manner, often in response to identified service gaps rather than based on predictive planning [10].
- **Paramedical Intervention Team (PIT)** units are assigned on the basis of existing hospital network structures, typically one per network [10].
- **Mobile Urgency Group (MUG)** units are generally allocated according to a population ratio, approximately one unit per 150,000 inhabitants, with a few exceptions [11].

Belgium's lack of a unified EMS location model prevents policymakers from implementing an optimal EMS resource allocation. Not only do these simple models lack the capability to consider the geographic, demographic and socio-economic variables that influence EMS demand, but they completely lack the ability to take the other tiers into account. This lack of integration between these tiers will further impede an effective resource allocation and will inevitably result in a suboptimal baseline and crisis response. And while this research focuses specifically on the asset reallocation phase during a MCI, the insights produced will be valuable to implement a unified model when the policy makers deem it appropriate. In general, this study aims to provide policymakers and emergency planners with insights to optimize EMS deployment and improve surge capacity.

1.3 Research Questions and Objectives

This thesis investigates the potential value of existing facility-location models when applied to the specific challenge of identifying which subset of Belgian EMS assets should be reallocated from baseline to surge EMS during a MCI. Rather than developing an entirely new model, this research focuses on identifying specific demand-influencing variables that may impact the applicability of a location model and suggesting several established models that might prove useful within this specific context.

The research centers on four key questions:

1. What are the strengths and limitations of deterministic, probabilistic, and dynamic location models when applied to surge EMS planning?
2. To what extent can existing location models support the identification of a subset of EMS assets for reallocation, while minimizing disruption to baseline service coverage?
3. What practical considerations, such as data availability, institutional constraints or computational complexity, must be accounted for when applying these models in the Belgian EMS context?
4. Which regional, demographic, and operational variables specific to Belgium influence EMS demand and could thus impact the applicability of the location models?

By addressing these questions, this research aims to provide policymakers with a thorough understanding to allow for the implementation of a theoretical framework-supported EMS reallocation. The ultimate objective is to enhance Belgium's surge response capabilities while preserving essential baseline EMS operations, thereby improving overall system resilience and patient outcomes during both regular operations and MCIs.

1.4 Thesis Outline

This thesis is structured to guide the reader progressively from foundational concepts to applied simulation, ensuring a comprehensive understanding of EMS location modeling in the context of MCIs in Belgium.

- **Chapter 1** introduces the context of EMS during MCIs, outlines the Belgian system's specific characteristics and limitations, and formulates the research objectives and questions.
- **Chapter 2** details the methodological approach, including the design of the scoping literature review and the framework for evaluating selected models through simulation.
- **Chapters 3 through 5** each explore a distinct category of location models:
  - Chapter 3 reviews **deterministic** models, focusing on those with fixed inputs and predictable outcomes.
  - Chapter 4 analyzes **probabilistic** models that incorporate uncertainty in ambulance availability and demand.
  - Chapter 5 discusses **dynamic** models and assesses their potential and limitations within the Belgian EMS context.
- **Chapter 6** presents the scenario-based simulation, applying selected models to realistic operational settings derived from Belgian data, and evaluates performance across various crisis scenarios.
- **Chapter 7** synthesizes the simulation results, compares model performance using defined metrics, and formulates practical insights and recommendations for EMS asset reallocation planning.

2

Methodology

This chapter outlines the approach used to evaluate the applicability of location theory models on the selection process of Surge Emergency Medical Services in countries that opted to share assets between regular and surge EMS.Rather than attempting to identify all relevant models, the goal of this thesis is to highlight some high-potential candidate models and demonstrate their practical potential. Therefore, a scoping review was first conducted to identify these high-potential models, followed by a scenario-based simulation to assess their performance. This approach is similar to previous research that attempted to evaluate the practical potential of location models [12].

2.1 Scoping literature review

2.1.1 Identifying relevant research

To evaluate the possible implementation of location models outside of their intended scope, a scoping review design was selected to identify models with potential while excluding models that are unlikely to work in this new context or that are too similar to already selected models. During this review, the academic database PubMed ((through Vesalius, the online library of the FPS Health, Food Chain Safety and Environment) and the academic search engine Google Scholar were used. Additionally, by using the snowball method, additional relevant research was identified.

A search algorithm was used in PubMed to identify potentially relevant literature related to ambulances or EMS in combination with theoretical models, geographical information system (GIS), asset management or resource allocation.

```
("Ambulances"[Mesh] OR "Emergency Medical Services"[Mesh]) AND ("Models, Theoretical"[Mesh] OR
↪ ("Geographic Information Systems"[Mesh] OR "asset management"[Title/Abstract] OR "resource
↪ allocation"[Title/Abstract])) NOT ("Triage"[Mesh] OR "Triage"[Title/Abstract] OR
↪ "AED"[Title/Abstract] OR "Diseases Category"[Mesh])
```

This algorithm searched for articles that include the medical subject headings (MeSH) terms "Ambulances" or "Emergency Medical Services", to leverage the platforms powerful capabilities. It further refined the results by only including articles that mention "Models, Theoretical" in MeSH or "Geographic Information Systems," "asset management," or "resource allocation" in the title or abstract. Since a large amount of results were focused on triage, automated external defibrillators (AED) or specific disease categories, the algorithm was refined using the "NOT"-operator. As Google Scholar lacks the MeSH capabilities used in the previous search algorithm, a slightly modified variant was used there:

```
(("Ambulances"OR"Emergency Medical Services") AND ("Geographic Information Systems" OR "Models"
↪ OR "asset management" OR "resource allocation")) NOT "Triage" NOT "AED"
```

An overview of the literature process can be viewed in 2.1.

The scoping review yielded 2,736 results from PubMed and 766 from Google Scholar. After removing duplicates, 3,257 unique records remained. Screening the titles for relevance resulted in 1,522 records that could be considered potentially relevant. To ensure completeness, records with unclear relevance were retained for the next stage. For 868 records, the full-text retrieval was unsuccessful, leading to 654 successfully retrieved publications. Of these 654 publications, 420 were excluded due to a lack of relevance. Citation tracking and snowball method yielded an additional 46 records from which 36 relevant full-texts were retrieved. This process resulted in a final dataset of 268 studies to be included in further analysis. Most of these focused on a specific location model or adapting an existing model within the EMS context of a specific country.

This broad collection of publications reflects the diverse range of theoretical models, but it is beyond the scope of this thesis to evaluate each of these models in depth. The aim is to identify a subset of models with a high potential for implementation in the Belgian surge EMS system. To ensure scientific rigor, a systematic selection procedure was used to identify the most promising candidates.

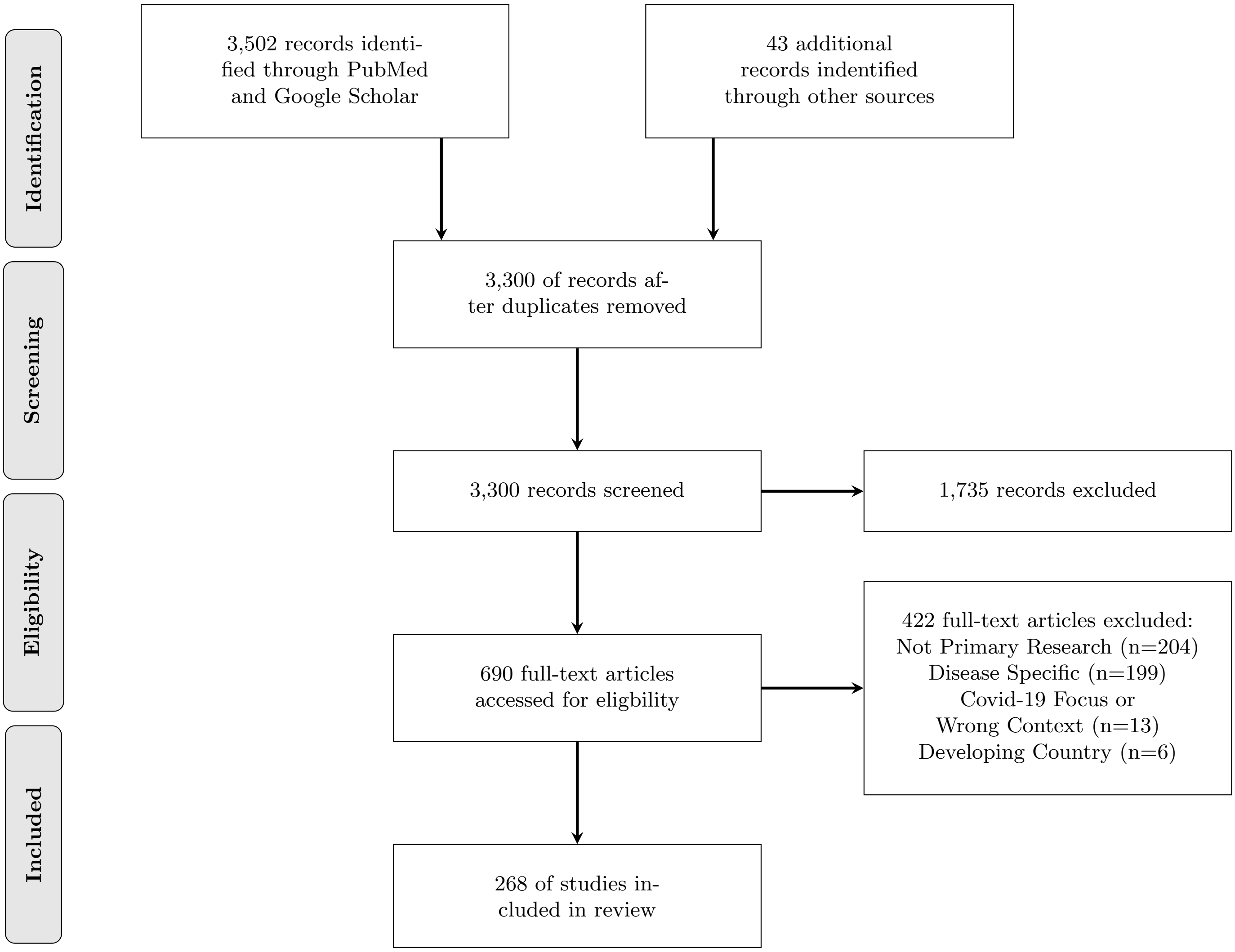


Figure 2.1: PRISMA flowchart

2.1.2 Selection criteria and Filtering process

The publications were analyzed chronologically based on their model development, rather than on publication date. This approach allowed for a better understanding of theoretical evaluations, derivative models and inter-model dependencies. Models that did not meet the key relevance criteria were excluded along with derivative models that failed to introduce a significant innovation or improvement. However, models that explicitly addressed limitations of their predecessors or demonstrated methodological advancements were retained if they were deemed relevant in the Belgian context. This process allowed for a streamlined processing of the publications by minimizing redundancy and focusing attention on models that offer a distinct analytical value.

The central filtering criteria was novelty. Models that only differ in parameter variations of other frameworks, without significant innovations were excluded. While such variations may improve results when applied in a specific setting, they lack generalizable value and do not have a meaningful contribution to the research question.

Relevance when applied in the Belgian EMS context was another decisive factor. Some theoretical models that seemed to be promising, were excluded due to their structural incompatibilities with the Belgian EMS context. As an example, many of the dynamic models appear to have great promise but assume the real-time tracking of EMS-assets, a operational capability that is currently unavailable in Belgium. Implementing such models would be beneficial, but require significant financial and technical investments, rendering them unusable in the short-term.

Transparency was also a selection criteria as this thesis attempts to identify models that need to be implemented by policymakers. Therefore only models with clearly reported methodologies and explicit assumptions were retained. Models that rely on proprietary algorithms, restricted datasets or vague methodologies were excluded. This criteria ensures that all selected models can be validated, replicated and adapted to the Belgian setting.

Applying the above selection process resulted in 13 models that meet the criteria of relevance, innovation, feasibility and transparency. These models represent the most promising candidates for short-term implementation in the Belgian EMS system.

2.2 Simulation design and model evaluation framework

To assess the practical value of selected location models in supporting EMS asset reallocation during MCIs, a simulation-based evaluation was conducted. This simulation aimed to determine each model's ability to maintain baseline EMS coverage while accommodating surge demands.

The simulation relied on actual EMS asset availability data from 2024 and modeled response under five crisis scenarios, each designed to reflect realistic operational challenges. These scenarios varied by:

- Geographical context: rural vs. urban settings
- Surge intensity: aligned with different levels of Medical Intervention Plan (MIP) activation

Because patient-level outcome data were unavailable, performance was evaluated using proxy indicators commonly referenced in EMS literature. These were grouped into four categories:

- Demand covered within predefined response times
- Equity Metrics:
  - Gini coefficient, measuring fairness in geographic and demographic coverage distribution

To contextualize model performance, two heuristic dispatch methods, commonly used in Belgian EMS practice, were included as baseline comparators. These heuristics, though operationally simple, lack theoretical grounding and have not been empirically validated.

The central hypothesis is that formal optimization-based models derived from location theory will outperform these heuristics by improving service coverage, promoting equitable resource distribution, and offering computationally feasible solutions during high-stress scenarios.

3 Deterministic Models

Deterministic models are characterized by the assumption that all input parameters, e.g. demand, travel time, and facility availability, are fixed and known with certainty. They are most effective in stable environments where data reliability is high, such as urban planning scenarios where future population density can be reasonably estimated.

Deterministic models provide a structured and computationally efficient approach to location problems. These models assume that all input parameters, such as demand distribution, travel times, and resource availability, are fixed and known with certainty. As a result, they are particularly useful for long-term strategic planning, where historical data can provide reliable estimations of future demand.

In the context of EMS, deterministic models play a crucial role in optimizing the placement of emergency response units. By strategically locating ambulances, these models help policymakers determine the minimum number of ambulances needed to achieve a predefined coverage threshold, ensuring accessibility within a specified response time.

Several deterministic models have been developed to guide ambulance placement based on coverage objectives. One of the most fundamental models, the **Set Covering Problem (SCP)**, aims to minimize the number of stations required to ensure that all demand points are covered at least once [13]. While SCP prioritizes full coverage, the **Maximal Covering Location Problem (MCLP)** shifts the focus to maximizing the number of demand points covered when only a limited number of stations are available [14]. Building on this, the **Tandem Equipment Allocation Model (TEAM)** and its variant the **Facility-Location, Equipment-Emplacement Technique (FLEET)**, extend the MCLP by introducing multiple service types, where each service has its own station constraints [15].

Although these models serve as a solid foundation for strategic planning, they share the common limitation that they focus on single coverage. This means that if an ambulance is occupied, the demand points it was assigned to cover remain temporarily uncovered, which can be a major drawback in high-demand or unpredictable situations. To address this limitation, multiple coverage models have been developed to ensure that demand points can be covered by more than one ambulance. The **Modified Maximal Covering Location Model (MMCLM)** introduces a secondary objective aimed at maximizing the number of demand points covered multiple times. Two variations of this model that are relevant to EMS will be discussed: **BACOP1** ensures that a portion of the population is covered at least twice within the same coverage standard, while **BACOP2** balances single and multiple coverage through a weighted objective function [16].Similarly, the **Double Standard Model (DSM)** incorporates two travel time thresholds, prioritizing demand points that can be reached multiple times within a shorter time frame [17].

Some later models began to focus on providing backup coverage to ensure quality of care. Other models also focus on providing backup coverage. The **Capacitated MCLP with backup coverage** considers resource constraints while ensuring that demand points have redundant coverage [18, 19].

The following sections will explore each of these models in detail, discussing their strengths, limitations, and practical applications.

3.1 Location Set Covering Problem (LSCP)

The Location Set Covering Problem (LSCP) is a deterministic facility location model introduced in 1971 by Toregas et al. [13]. The LSCP aims to minimize the number of facilities required while still ensuring that all demand points are covered within a predefined service distance or response time.

In the EMS context, the LSCP is useful for determining the baseline number of ambulance stations needed to ensure that every emergency demand location is within an acceptable response time.

Like all discrete models, the LSCP makes some key assumptions to ensure computable efficiency and solvability, especially by the means that were available in 1971:

- There exists a static demand pattern, represented by a **fixed set of demand points**.
- There exists a **predefined maximum response time** (or in absence, a distance threshold) that determines coverage feasibility.
- Facilities can only be located at **discrete candidate locations** (e.g., hospitals, fire stations).
- The objective is **binary coverage**; a demand point is either covered within the predefined maximum response time or not, there does not exist a partial coverage.

3.1.1 Mathematical Formulation

Decision Variable

$$Y_j = \begin{cases} 1, & \text{if the demand point is covered within the predefined maximum response time} \\ 0, & \text{otherwise} \end{cases}$$

Sets and Parameters

- I = Set of demand points (indexed by i)).
- J = Set of candidate facility locations (indexed by j).
- S_i = Set of facilities j that can cover demand point i within the maximum response time.

Objective Function

$$\textbf{Minimize} \quad \sum_{j \in J} Y_j \tag{1}$$

The LSCP aims to minimize the number of facilities.

Coverage Constraints

$$\sum_{j \in N_i} Y_j \geq 1, \quad \forall i \in I \tag{2}$$

This constraint ensures that each demand point i is covered by at least one facility within the maximum response time.

Binary Constraints

$$Y_j \in \{0, 1\}, \quad \forall j \in J \tag{3}$$

This constraint enforces that a facility is either placed at a location j (($x_j = 1$) or not ($X_j = 0$).

3.1.2 Example application in EMS

Consider an urban region with **six demand points** and **three candidate ambulance stations**. The goal is to determine the **minimum number of stations** required so that all demand points are covered within **12 minutes**.

To solve this problem, one could start by creating a travel time matrix $A = a_{ij}$ where i represents each intervention, j represents each ambulance service and a_{ij} equals the travel time between the intervention site and the station.

$$A = \begin{bmatrix} 11 & 4 & 15 & 17 & 21 & 16 \\ 10 & 16 & 6 & 10 & 2 & 12 \\ 16 & 8 & 9 & 11 & 12 & 18 \end{bmatrix}$$

Using the 12-minute threshold, we derive a binary **binary accessibility matrix** $B = [b_{ij}]$, where:

$$b_{ij} = \begin{cases} 1, & \text{if } a_{ij} \leq 12 \\ 0, & \text{otherwise} \end{cases}$$

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Demand point i	**Candidate facility locations covering i**
1	$\{A, B\}$
2	$\{A, C\}$
3	$\{B, C\}$
4	$\{B, C\}$
5	$\{B\}$
6	$\{B\}$

Table 3.1: LSCP facility coverage table.

Based on the table, two combinations of facilities, $\{A, B\}$ and $\{A, C\}$, are sufficient to cover all six demand points with the minimum number of stations.

3.1.3 Strengths and Limitations

The LSCP offers a **minimum-cost solution** to achieve complete coverage. Its simplicity and computational efficiency make it particularly useful for **strategic EMS planning** in static or predictable environments. For example, it can be used in pre-crisis planning to determine the optimal number and location of stations, or in real-time to identify the minimal operational resources needed to maintain baseline coverage during a crisis.

However, the LSCP also has notable limitations:

- It assumes **binary coverage**, which does not reflect the variability of real-world response times.
- It does **not account for facility capacity**, such as ambulance availability or simultaneous calls.
- It ignores **dynamic demand patterns** and temporal fluctuations in incident frequency.

Despite these limitations, the LSCP remains a foundational model in emergency service location planning and provides a valuable benchmark for more advanced models.

3.2 Maximal Covering Location Problem (MCLP)

The Maximal Covering Location Problem (MCLP), introduced by Church and ReVelle (1974), extends the Location Set Covering Problem (LSCP) by addressing situations where not all demand points can be fully covered due to resource constraints. Instead of requiring that all demand points be covered, the MCLP maximizes the number of demand points covered given a fixed number of facilities [14].

In the EMS context, the MCLP is useful for determining the best allocation of a limited number of ambulances or EMS stations to cover the largest possible population within a predefined response time (e.g., 12 minutes in Belgium). This is particularly relevant in crisis management when the number of EMS units is insufficient to provide full coverage, and decision-makers must prioritize high-demand areas.

Like all discrete models, the MCLP makes some key assumptions to ensure computational efficiency and solvability:

- There exists a static demand pattern represented by a **fixed set of demand points**.
- A **fixed number of facilities** can be placed.
- A predefined **maximum response time** (or distance threshold) that determines whether a demand point is covered.
- **No requirement for full coverage**, thus not every demand point needs to be covered

3.2.1 Mathematical Formulation

Decision Variable

$$x_j = \begin{cases} 1, \text{if a facility is located at site } j \\ 0, \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, \text{if a demand point } i \text{ is covered} \\ 0, \text{otherwise} \end{cases}$$

Sets and Parameters

- I = Set of demand points (indexed by i).
- J = Set of candidate facility locations (indexed by j).
- S_i = Set of facilities j that can cover demand point i within the maximum response time.
- d_i = Demand weight at demand point i
- P = The fixed number of facilities available

Objective Function

$$\textbf{Maximize} \sum_{i \in I} d_i y_i \tag{1}$$

The goal of MCLP is to **maximize the total covered demand** by the selected facilities.

Coverage Constraints

$$y_i \leq \sum_{j \in S_i} x_j, \quad \forall i \in I \tag{2}$$

This constraint ensures that each demand point i is only considered as covered ($y_{=}1$) if at least one facility in S_i is selected.

Facility Constraints

$$\sum_{j \in J} x_j \leq P \tag{3}$$

This ensures that no more than P facilities are selected.

Binary Constraints

$$x_j, y_i \in 0, 1, \quad \forall j \in J, \forall i \in I \tag{4}$$

This enforces that a facility can only be placed ($y_i = 1$) or not ($y_i = 0$), and that the demand node i, is either considered covered ($x_j = 1$) or not ($x_j = 0$).

3.2.2 Example application in EMS

Consider an urban region with **six demand points** and **two candidate ambulance stations**. The goal is to determine the **maximum** total population covered within **12 minutes**. To solve this problem, one could start by creating a travel time matrix $A = a_{ij}$ where i represents each intervention, j represents each ambulance service and a_{ij} equals the travel time between the intervention site and the station.

$$A = \begin{bmatrix} 11 & 4 & 15 & 17 & 21 & 16 \\ 10 & 16 & 6 & 10 & 2 & 12 \\ 16 & 8 & 9 & 11 & 12 & 18 \end{bmatrix}$$

Using the aforementioned cutoff time of 12 minutes, a binary accessibility matrix B can be created where $b_{ij} = 1, \text{if } a_{ij} \leq 12$ or $b_{ij} = 0, \text{if } a_{ij} > 12$:

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Now consider the following population weights d_i as presented in table below.

Demand point i	**Population** d_i	**Candidate facility locations covering** i
1	10.000	$\{A, B\}$
2	15.000	$\{A, C\}$
3	12.000	$\{B, C\}$
4	8.000	$\{B, C\}$
5	9.000	$\{B\}$
6	5.000	$\{B\}$

Table 3.2: MCLP population weights and facility coverage table

By using the information in table 3.2, we find that candidate facility A covers a population of 25.000, B a population of 44.000 and C a population of 35.000. Since we are aiming to maximize coverage with only 2 locations, $\{B, C\}$ would be the most efficient set of locations.

3.2.3 Strengths and Limitations

The MCLP can provide a **maximal covering** solution that takes the resource constraints into account. It offers a practical decision-making tool for policymakers, particularly in budget-constrained or crisis management scenarios. The flexibility in weighting demands allows the decision-makers to choose their prioritization. However, the MCLP also has limitations: **demand weights** may be difficult to estimate, and it remains a static model that does not account for real-time demand fluctuations.

3.3 Tandem Equipment Allocation Model (TEAM)

The Tandem Equipment Allocation Model (TEAM), introduced by Schilling et al. in 1979, was developed to address a multi-tiered service system, where different types of equipment or personnel must be allocated. Unlike the previous models, TEAM accounts for the simultaneous placing and allocation of both primary assets and more specialized equipment and/or personnel. This model assumes that the specialized assets can only be located on the same sites as the primary assets [15].

The differentiation between assets in this model, is particularly relevant for Emergency Medical Services (EMS) in Belgium, where the three-tiered EMS system (Ambulance, PIT, and MUG) requires coordination between different types of emergency units.

Like all discrete models, the MCLP makes some key assumptions to ensure computational efficiency and solvability:

- There exists a static demand pattern represented by a **fixed set of demand points**.
- There exists **fixed set of candidate locations for primary resources**
- There exists **fixed set of candidate locations for specialized resources**
- There exists a **predefined maximum response time** (or in absence, a distance threshold) that determines coverage feasibility for both primary and specialized resources.
- The objectives are **binary coverage**; a demand point is either covered within the predefined maximum response time or not, partial coverage is not allowed.
- There exists **a fixed amount of primary and specialized equipment** that need to be allocated.

3.3.1 Mathematical Formulation

Decision Variable

$$x_j^p = \begin{cases} 1, \text{if a primary equipment is located at site } j \\ 0, \text{otherwise} \end{cases}$$

$$x_j^s = \begin{cases} 1, \text{if a specialized equipment is located at site } j \\ 0, \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, \text{if a demand point } i \text{ is covered by both primary and specialized equipment} \\ 0, \text{otherwise} \end{cases}$$

Sets and Parameters

- I = Set of demand points (indexed by i)).
- J = Set of candidate facility locations (indexed by j).
- p^p = The number of primary equipment units to be located.
- p^s = The number of specialized equipment units to be located.
- S^p = The service distance standard for primary equipment.
- S^s = The service distance standard for specialized equipment.

Objective Function

$$\textbf{Maximize} \quad \sum_{i \in I} a_i y_i \tag{1}$$

The TEAM aims to **maximize the total demand** covered by the selected facilities.

Coverage Constraints

$$\sum_{j \in N_j^p} X_j^p \geq y_i, \quad \forall i \in I \tag{2}$$

$$\sum_{j \in N_j^s} x_j^s \geq y_i, \quad \forall i \in I \tag{3}$$

A demand node is only covered if it is covered by both primary and specialized equipment.

$$\sum_{j \in J} x_j^p = p^p \tag{4}$$

$$\sum_{j \in J} x_j^s = p^s \tag{5}$$

The total number of primary and specialized equipment units **must not exceed their respective fixed quantities**.

Binary Constraints

$$x_j^p, x_j^s = 0, 1, \quad \forall i \in I \tag{6}$$

$$y_i = 0, 1, \quad \forall i \in I \tag{7}$$

These constraints enforce that:

- A facility can only be placed ($y_i = 1$) or not ($y_i = 0$).
- The node i, can only be considered covered by primary equipment units ($x_j^p = 1$) or not ($x_j^p = 0$).
- The node i, can only be considered covered by specialized equipment units ($x_j^s = 1$) or not ($x_j^s = 0$).

Dependency Constraints

$$x_j^p \leq x_j^s, \quad \forall i \in I \tag{8}$$

This constraint enforces that specialized equipment can only be located at nodes that also contain primary equipment (hence the word "tandem" in the model name).

3.3.2 Example application in EMS

Consider an urban region with **six demand points** and **three candidate locations**. The goal is to **maximize the total population covered within a response time of 12 minutes with primary equipment and 15 minutes with specialized equipment**, using **two primary** and **one specialized** equipment units. To solve this problem, one could start by creating a travel matrix $A^p = a_{ij}^p$ where i represents each intervention, j represents each equipment location and a_{ij}^p equals the travel time between the intervention site and the station for primary equipment. Similarly, a second travel matrix $A^s = a_{ij}^s$ can be constructed for the specialized equipment.

$$A^s = \begin{bmatrix} 11 & 4 & 15 & 17 & 21 & 16 \\ 10 & 16 & 6 & 10 & 2 & 12 \\ 16 & 8 & 9 & 11 & 12 & 18 \end{bmatrix}$$

$$A^p = \begin{bmatrix} 6 & 12 & 13 & 2 & 4 & 9 \\ 1 & 38 & 21 & 11 & 9 & 7 \\ 10 & 6 & 16 & 14 & 3 & 12 \end{bmatrix}$$

Using the aforementioned cutoff time of 12 minutes, two binary accessibility matrices can be created:

- B^p where $b_{ij}^p = 1, \text{if } a_{ij}^p \leq 12$ or $b_{ij}^p = 0, \text{if } a_{ij}^p > 12$.
- B^s where $b_{ij}^s = 1, \text{if } a_{ij}^s \leq 15$ or $b_{ij}^s = 0, \text{if } a_{ij}^s > 15$.

$$B^p = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$B^s = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Now consider the following population weights d_i as presented in the table below.

		Suitable facility locations	
Demand point i	**Population d_i**	**Primary equipment**	**Specialized equipment**
1	10.000	$\{A, B\}$	$\{A, B, C\}$
2	15.000	$\{A, C\}$	$\{A, C\}$
3	12.000	$\{B, C\}$	$\{A\}$
4	8.000	$\{B, C\}$	$\{A, B, C\}$
5	9.000	$\{B\}$	$\{A, B, C\}$
6	5.000	$\{B\}$	$\{A, B, C\}$

Table 3.3: TEAM population weights and facility coverage table

The TEAM model first selects the locations that maximizes the demand coverage by the primary equipment. Therefore, the 2 primary equipment locations will be located at B, and C, which cover a population of 44.000 and 35.000 respectively. As this model has the dependency constraint that an specialized equipment can only be located at nodes that also contain primary equipment, location A, although covering all population with specialized equipment, is no longer being a valid location due to the lack of primary equipment. Therefore the location C, which covers a population of 47.000 will be selected. The TEAM model, given this example will select locations B and C for the primary equipment and selects location C for the specialized equipment location.

3.3.3 Strengths and Limitations

The TEAM model provides a **hierarchical maximal covering** solution, with asset differentiation, that allows for the prioritization of high-demand areas while under resource constraints. It offers a more realistic approach as full coverage (required by other models like LSCP) is often not feasible. Its flexible weighing constraint allows for the decision-makers to chose their priorities and while subjective weights can be used, it is advisable to use objective weights even when they are sometimes difficult to estimate. However, it still remains a static model and does not take the real-time demand into account.

3.4 Facility-Location, Equipment-Emplacement Technique (FLEET)

In the same publication as the previously discussed TEAM model, Schilling et al. also introduced a variant that they named the Facility-Location, Equipment-Emplacement Technique (FLEET) [15]. The FLEET model differs from the TEAM model by removing the constraint that specialized equipment can only be located at sites where primary equipment is placed.

In the EMS context, the FLEET is useful for quickly getting a solution that balances coverage requirements with resource constraints. By removing the constraint that specialized equipment can only be placed on nodes that already contain primary equipment, it might better reflect EMS in some countries.

Like all discrete models, the FLEET makes some key assumptions

- There exists a static demand pattern represented by a **fixed set of demand points**.
- There exists **fixed set of candidate locations for primary resources**
- There exists **fixed set of candidate locations for specialized resources**
- There exists a **predefined maximum response time** (or in absence, a distance threshold) that determines coverage feasibility for both primary and specialized resources.
- The objectives are **binary coverage**; a demand point is either covered within the predefined maximum response time or not, there does not exist a partial coverage.
- There exists **a fixed amount of primary and specialized equipment** that need to be allocated.

3.4.1 Mathematical Formulation

Decision Variables

$$x_j^p = \begin{cases} 1, \text{if a primary equipment is located at site } j \\ 0, \text{otherwise} \end{cases}$$

$$x_j^s = \begin{cases} 1, \text{if a specialized equipment is located at site } j \\ 0, \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, \text{if a demand point } i \text{ is covered by both primary and specialized equipment} \\ 0, \text{otherwise} \end{cases}$$

Sets and Parameters

- I = Set of demand points (indexed by i)).
- J = Set of candidate facility locations (indexed by j).
- p^p = The number of primary equipment units to be located.
- p^s = The number of specialized equipment units to be located.
- S^p = The service distance standard for primary equipment.
- S^s = The service distance standard for specialized equipment.

Objective Function

$$\textbf{Maximize} \quad \sum_{i \in I} a_i y_i \tag{1}$$

The FLEET aims to maximize the total covered demand by the selected facilities.

Coverage Constraints

$$\sum_{j \in N_j^p} X_j^p \geq y_i, \quad \forall i \in I \tag{2}$$

$$\sum_{j \in N_j^s} x_j^s \geq y_i, \quad \forall i \in I \tag{3}$$

A demand node is only covered if it is covered by both primary and specialized equipment.

$$\sum_{j \in J} X_j^p = p^p \tag{4}$$

$$\sum_{j \in J} x_j^s = p^s \tag{5}$$

The sum of all primary equipment units and all specialized equipment units may never exceed their predefined respective fixed amounts.

Binary Constraints

$$x_j^p, x_j^s = 0, 1, \quad \forall i \in I \tag{6}$$

$$y_i = 0, 1, \quad \forall i \in I \tag{7}$$

These constraints enforce that:

- A facility can only be placed ($y_i = 1$) or not ($y_i = 0$).
- The node i, can only be considered covered by primary equipment units ($x_j^p = 1$) or not ($x_j^p = 0$).
- The node i, can only be considered covered by specialized equipment units ($x_j^s = 1$) or not ($x_j^s = 0$).

3.4.2 Example application in EMS

We reprise the same example as before with the TEAM model: Consider an urban region with **six demand points** and **three candidate locations**. The goal is to **maximize the total population covered within a response time of 12 minutes with primary equipment and 15 minutes with specialized equipment**, using the two primary and a single specialized equipment units. To solve this problem, one could start by creating a travel matrix $A^p = a_{ij}^p$ where i represents each intervention, j represents each equipment location and a_{ij}^p equals the travel time between the intervention site and the station for primary equipment. Similarly, a second travel matrix $A^s = a_{ij}^s$ can be constructed for the specialized equipment.

$$A^s = \begin{bmatrix} 11 & 4 & 15 & 17 & 21 & 16 \\ 10 & 16 & 6 & 10 & 2 & 12 \\ 16 & 8 & 9 & 11 & 12 & 18 \end{bmatrix}$$

$$A^p = \begin{bmatrix} 6 & 12 & 13 & 2 & 4 & 9 \\ 1 & 38 & 21 & 11 & 9 & 7 \\ 10 & 6 & 16 & 14 & 3 & 12 \end{bmatrix}$$

Using the aforementioned cutoff time of 12 minutes, two binary accessibility matrices can be created:

- B^p where $b_{ij}^p = 1, \text{if } a_{ij}^p \leq 12 \text{ or } b_{ij}^p = 0, \text{if } a_{ij}^p > 12$.
- B^s where $b_{ij}^s = 1, \text{if } a_{ij}^s \leq 15 \text{ or } b_{ij}^s = 0, \text{if } a_{ij}^s > 15$.

$$B^p = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$B^s = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Now consider the following population weights d_i as presented in table 3.4.

		Suitable facility locations	
Demand point i	**Population d_i**	**Primary equipment**	**Specialized equipment**
1	10.000	$\{A, B\}$	$\{A, B, C\}$
2	15.000	$\{A, C\}$	$\{A, C\}$
3	12.000	$\{B, C\}$	$\{A\}$
4	8.000	$\{B, C\}$	$\{A, B, C\}$
5	9.000	$\{B\}$	$\{A, B, C\}$
6	5.000	$\{B\}$	$\{A, B, C\}$

Table 3.4: FLEET population weights and facility coverage table

The FlEET model selects the locations which maximizes the demand coverage by the primary equipment and specialized equipment independently. Therefore, the 2 primary equipment locations will be located at B, and C, who cover a population of 44.000 and 35.000 respectively. As location C covers the entire population within the maximum allowed travel time of 15 minutes, it will be located at this location.

3.4.3 Strengths and Limitations

The FLEET model allows for independent placement of primary and specialized equipment, which is especially relevant for simulating multi-tiered EMS systems. While this flexibility is advantageous, FLEET also has limitations:

- As a deterministic model, it does not account for ambulance availability uncertainty.
- Its assumption of fixed resource pools limits adaptability during demand surges or fleet expansion.
- Its use of binary coverage oversimplifies real-world EMS reliability.

3.5 Maximal Backup Coverage Problem (BACOP I and II)

In none of the previous models was the unavailability of an asset during its coverage explicitly modeled. One of the first attempts to include this aspect into a location model was made by Daskin and Stern in 1981 when they created the Hierarchical Objective Set Covering Model (HOSCM) [20]. Another variant, the BACOP (Backup Coverage Optimization Problem) models were developed by Hogan and ReVelle in 1986 to address the limitations of single coverage deterministic models in emergency service location planning [16]. These models were created to enhance the reliability and effectiveness of emergency response systems by introducing the concept of backup coverage. These BACOP models recognize that having only one vehicle available to cover a demand zone may not be sufficient, as that vehicle could be busy when needed. BACOP I aims to maximize the population with both primary and backup coverage while operating under resource constraints (fixed amount of assets). BACOP II allows for flexibility for the decision-makers as the weight that is given between primary and backup coverage can be determined. It provides insights into trade-offs between ensuring broad access (primary coverage) versus enhancing reliability through redundancy (backup-coverage).

In the EMS context, the BACOP models are useful as they aim to increase the likelihood of having a vehicle available to respond to emergencies, thus improving the overall performance and resilience of emergency medical services (EMS) systems. BACOP I focuses on maximizing the population with both primary and backup coverage. It aims to ensure that as many people as possible have access to both a primary and a secondary service or facility.

Like all discrete models, both the BACOP models make some key assumptions

- There exists a static demand pattern represented by a **fixed set of demand points**.
- There exists a **fixed set of resources**.
- There exists a **predefined maximum response time** (or in absence, a distance threshold) that determines coverage feasibility.
- Facilities can only be placed at **discrete candidate locations** (e.g. hospitals, fire stations).
- **Backup coverage** exists only when at least two servers are located such that they can respond within the maximum response time.

3.5.1 Mathematical Formulation BACOP I

Decision Variable

$$x_j = \begin{cases} 1, \text{if a facility is located at site } j \\ 0, \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, \text{if a demand point } i \text{ is covered} \\ 0, \text{otherwise} \end{cases}$$

$$u_i = \begin{cases} 1, \ \text{if demand node } i \text{ is covered twice} \\ 0, \ \text{otherwise} \end{cases}$$

Sets and Parameters

- I = Set of demand points (indexed by i).
- J = Set of candidate facility locations (indexed by j).
- x_j = Integer number of facilities located at candidate facility location j.
- a_i = Population at demand node i.
- N_i = Set of candidate locations with a response time to demand i within the maximum primary response time.
- M_i = Set of candidate location with a response time to demand i within the maximum backup response time.
- p = Total number of facilities.

Objective Function

$$\textbf{Maximize} \sum_{i \in I} a_i u_i \tag{1}$$

The BACOP I aims to maximize the total population (weighted by a_i) that has backup coverage. The goal is to prioritize redundancy, ensuring that as many people as possible have access to at least two facilities.

Coverage Constraints

$$\sum_{j \in N_i} x_j - u_i \geq 1, \quad \forall i \in I \tag{2}$$

$$\sum_{j \in M_i} x_j \geq 1, \quad \forall i \in I \tag{3}$$

The first constraint ensures backup coverage, and the second ensures at least one facility provides primary coverage.

Facility Constraints

$$\sum_{j \in J} x_j = P \tag{4}$$

This constraint ensures that P facilities are selected.

Binary Constraints

$$u_i, y_i, x_j \in 0, 1, \quad \forall j \in J, \forall i \in I \tag{5}$$

This enforces that a facility can only be placed ($x_j = 1$) or not ($x_j = 0$), that the node i can be covered by primary coverage ($y_i = 1$) or not ($y_i = 0$) and that the node i can be covered by both primary and backup coverage ($u_i = 1$) or not ($u_i = 0$) .

3.5.2 Mathematical Formulation BACOP II

Decision Variable

$$x_j = \begin{cases} 1, \text{if a facility is located at site } j \\ 0, \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, \text{if a demand point } i \text{ is covered} \\ 0, \text{otherwise} \end{cases}$$

$$u_i = \begin{cases} 1, \ \text{if demand node } i \text{ is covered twice} \\ 0, \ \text{otherwise} \end{cases}$$

Sets and Parameters

- I = Set of demand points (indexed by i).
- J = Set of candidate facility locations (indexed by j).
- x_j = Integer number of facilities located at candidate facility location j.
- a_i = Population at demand node i.
- N_i = Set of candidate locations with a response time to demand i within the maximum primary response time.
- M_i = Set of candidate location with a response time to demand i within the maximum backup response time.
- p = Total number of facilities.
- θ = A weight chosen between 0 and 1 that determines the value distribution between primary and backup coverage.

Objective Function

$$\textbf{Maximize} \quad \theta \sum_{i \in I} a_i y_i + (1 - \theta) \sum_{i \in I} a_i u_i \tag{1}$$

The BACOP II aims to maximize the total covered demand by the selected facilities, but no longer requires coverage of each demand node. Alternatively, the objective function can be written as a bi-objective formulation:

$$\textbf{Maximize} \quad Z_1 = \sum_{i \in I} a_i y_i$$

$$\textbf{Maximize} \quad Z_2 = \sum_{i \in I} a_i u_i$$

Z_1 represents the total population covered by at least one facility. It prioritizes ensuring that all demand points have basic access to services. Z_2 represents the population covered by at least two facilities. It emphasizes redundancy to improve reliability.

Coverage Constraints

$$\sum_{j \in N_i} x_j - y_i - u_i \geq 0, \quad \forall i \in I \tag{2}$$

This constraint ensures primary coverage, and $u_i = 1$ only when at least two facilities cover demand point i.

Facility Constraints

$$\sum_{j \in J} x_j = P \tag{3}$$

This constraint ensures that P facilities are selected.

Binary Constraints

$$u_i, y_i \in 0, 1, \quad \forall j \in J, \forall i \in I \tag{4}$$

This enforces that a facility can only be placed ($y_i = 1$) or not ($y_i = 0$) and that the node i can only be considered covered ($u_i = 1$) or not ($u_i = 0$).

3.5.3 Example application in EMS

Consider an area with **six demand points** and **three candidate facility locations**. To solve this problem, one could start by creating a travel time matrix $A = a_{ij}$ where i represents each demand point, j represents each ambulance service and a_{ij} equals the travel time between the intervention site and the station.

$$A = \begin{bmatrix} 4 & 8 & 12 & 17 & 21 & 16 \\ 14 & 13 & 6 & 10 & 2 & 18 \\ 16 & 13 & 21 & 18 & 4 & 2 \end{bmatrix}$$

Using the aforementioned cutoff time of 12 minutes, a binary accessibility matrix B can be created where $b_{ij} = 1$, if $a_{ij} \leq 12$ or $b_{ij} = 0$, if $a_{ij} > 12$:

$$B = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Now consider the following population weights d_i as shown in the table below.

Demand point i	**Population d_i**	**Candidate facility locations covering i**
1	100	$\{A\}$
2	200	$\{A\}$
3	150	$\{A, B\}$
4	120	$\{B\}$
5	180	$\{B, C\}$
6	250	$\{C\}$

Table 3.5: BACOP population weights and facility coverage table

3.5.4 Strengths and Limitations

The BACOP models (BACOP I and BACOP II) offer significant strengths in the context of Emergency Medical Services (EMS) localization. One of their primary advantages is the explicit incorporation of backup coverage, which enhances system reliability by ensuring service to demand points even when the primary facility is occupied. This is particularly critical in EMS scenarios where delays or failures in response can have severe consequences. BACOP I focuses on maximizing the population with both primary and backup coverage, making it ideal for systems prioritizing redundancy and reliability. BACOP II, on the other hand, introduces a bi-objective framework that balances first and backup coverage, allowing decision-makers to optimize for both accessibility and resilience based on specific priorities. Additionally, these models are computationally efficient compared to more complex stochastic or dynamic models, which enhances their practicality for real-world EMS planning.

However, the BACOP models have several limitations when applied to EMS localization. They rely on deterministic assumptions about coverage areas and fail to account for uncertainties such as varying traffic conditions, dynamic demand patterns, or facility failures during disasters. While backup coverage improves reliability, it does not explicitly model probabilistic scenarios where multiple facilities might simultaneously become unavailable due to large-scale emergencies. Additionally, the models assume uniform service capacity across facilities and do not account for variations in resource availability or operational constraints, such as differences in ambulance fleet sizes. As a result, while the BACOP models provide a solid foundation for EMS localization planning, they may need to be supplemented with stochastic or scenario-based approaches to fully address the complexities and uncertainties inherent in EMS systems.

3.6 Double Standard Model (DSM)

The Double Standard Model (DSM), introduced by Gendreau et al. in 1998, is a tactical-level ambulance location framework [21]. It is designed to optimize coverage under dual response time thresholds. This model introduced a novel approach to balancing stringent coverage requirements with operational feasibility, leveraging tabu search heuristics to solve complex combinatorial problems inherent in emergency service allocation. By prioritizing double coverage within a smaller radius while ensuring universal coverage within a larger radius, the DSM addressed gaps in earlier single-coverage models and provided a template for integrating hierarchical service standards into location planning.

Like all discrete models, the DSM makes some key assumptions:

- There exists a static demand pattern represented by a **fixed set of demand points**.
- There exists **two fixed coverage radii** which define two concentric service zones:
  - The inner zone requires dual ambulance coverage for a predefined proportion of demand points (usually population).
  - The outer zone requires single ambulance coverage for all demand points.
- Facilities can only be placed at **discrete candidate locations** (e.g. hospitals, fire stations).
- Only a certain number of ambulances can be placed, reflecting budget or resource constraints.
- Up to two ambulances can be located per station to avoid over concentration.
- The objective is **binary coverage**; a demand point is either covered within the predefined maximum response time or not, there is no partial coverage.

3.6.1 Mathematical Formulation

Decision Variable

$$y_j = \begin{cases} 2, \text{if two ambulances are located at the potential site } j \\ 1, \text{if a single ambulance is located at the potential site } j \\ 0, \text{otherwise} \end{cases}$$

$$x_i^1 = \begin{cases} 1, \text{if a demand point } i \text{ is covered at least once within radius } r_1 \\ 0, \text{otherwise} \end{cases}$$

$$x_i^2 = \begin{cases} 1, \text{ if demand node } i \text{ is covered twice within radius } r_2 \\ 0, \text{ otherwise} \end{cases}$$

Sets and Parameters

- V = Set of demand points (indexed by i).
- W = Set of candidate facility locations (indexed by j).
- λ_i = Demand intensity at point i.
- α = Minimum proportion of total demand requiring dual coverage within r_1.
- a_{ij} = 1 if demand point i is within r_1 of location j.
- δ_{ij} = 1 if demand point i is within r_2 of location j.

Objective Function

$$\textbf{Maximize} \quad \sum_{i \in V} \lambda_i x_i^2 \tag{1}$$

The DSM seeks to maximize the total demand covered by at least two ambulances within $r1$, this reflects the model's emphasis on redundancy, ensuring critical areas receive rapid dual responses.

Coverage Constraints

$$\sum_{i\in V} \lambda_{ij} y_j \geq 1 \quad \forall i \in V \tag{2}$$

$$\sum_{i\in V} \lambda_i (x_i^1 + x_i^2) \geq \alpha \sum_{i\in I} \lambda_i \tag{3}$$

$$\sum_{j\in W} a_{ij} y_j \geq x_i^1 + 2x_i^2 \quad \forall i \in V \tag{4}$$

$$x_i^2 \leq x_i^1 \quad \forall i \in V \tag{5}$$

These constraints ensure that every demand point must be within r_2 of at least one ambulance, at least α proportion of total demand must lie within r_i of one or more ambulances. These constraints also ensure that $x_i^2 = 1$ only if two ambulances cover i within r_1, and $x_i^1 = 1$ if at least one does. Constraint 4 ensures that a point i cannot be doubly covered unless it is singly covered.

Binary Constraints

$$x_i^1, x_i^2 \in 0, 1 \quad \forall i \in V, \forall j \in W \tag{6}$$

$$y_j \in 0, 1, 2 \quad \forall i \in V, \forall j \in W \tag{7}$$

This enforces that in a facility no ambulances ($y_j = 0$), one ambulance ($y_j = 1$) or two ambulances ($y_j = 2$) can be placed. Also a location can only be covered or not.

3.6.2 Example application in EMS

Consider an area with **8 demand points, four potential ambulance locations**, and **3 available ambulances**. To solve this example, one could start by creating a travel time matrix $A = a_{ij}$ where i represents the potential ambulance locations, j represents the demand points and a_{ij} equals the travel time between the potential ambulance locations and the demand points.

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 \end{bmatrix}$$

Given a cuttoff travel time for radius r_1 of 2 and for radius R_2 of 4, we can calculate the following coverage matrices, where a_{ij} represents the first and b_{ij} the second radii.

$$a_{ij} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$b_{ij} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

This DSM example can be solved through three distinct methodologies, each with unique advantages and limitations.

For small-scale instances (e.g., 8 demand points and 4 locations), manual calculation involves evaluating all possible ambulance allocations. For example, with 3 ambulances and 4 locations, there are $(4 + 3 - 1)/3 = 20$ possible distributions under a maximum of 2 ambulances per site. While feasible for trivial cases, this approach suffers from combinatorial explosion: doubling the problem size to 16 demand points and 8 locations would yield over 10,000 configurations. Manual methods also struggle with constraint verification (e.g., ensuring 80% dual coverage) and become error-prone even in moderately sized scenarios.

To solve this combinatorial explosion, Gendreau et al. proposed in 1997 a tabu search heuristic that iteratively explores neighboring solutions while maintaining a short-term memory (the so called "tabu list") to avoid revisiting already used configurations [21]. This method is able to handle the prioritization that was put forward by the DSM and can make more complex situations computable. In their original paper, Gendreau et al. demonstrated scalability, solving real-world instances with 100+ demand points in minutes. However, this method does not guarantee that the result of this heuristic will always lead to a global optimal solution.

With the advent of more powerful computers, a mixed-integer linear programming (MILP) method can also be used. The presented example Python code is an example of this approach and can guarantee a global optimal solution. As a downside, this method also faces an exponential time complexity. Doubling the number of demand points from 8 to 16 increases the possible combinations from 256 to 65,536. Thus for a large dataset, a hybrid approach should combine exact methods with heuristics—using MILP for small subproblems and tabu search for large-scale optimization.

3.6.3 Strengths and Limitations

The DSM introduced several advancements over prior models. The hierarchical coverage structure (r_1 and r_2) allowed policymakers to balance strict service standards with practical feasibility. Computationally, the integration of tabu search heuristics enabled efficient solutions for large-scale instances, overcoming limitations of exact methods. Additionally, the model's flexibility in ambulance allocation (permitting two units per site) mirrored real-world operational practices better than single-unit restrictions.

Despite its innovations, the DSM has notable limitations. Its static nature ignores temporal demand fluctuations and real-time ambulance availability, a gap addressed later by dynamic redeployment models. The deterministic travel time assumption oversimplifies urban mobility, neglecting probabilistic delays. Furthermore, the model does not incorporate queuing effects or busyness probabilities, which are critical for systems under high utilization. The fixed ambulance count p may also lead to infeasibility if coverage requirements are too stringent, necessitating manual parameter adjustments. Finally, while tabu search improves scalability, computational complexity remains challenging for larger networks.

The DSM's static assumptions are often relaxed in practice using dynamic redeployment strategies, which allows the integration of real-time data and machine learning for adaptive coverage. This transforms the model type to a dynamic model [21].

3.7 Potential and limitations of deterministic location models

It is evident that the original deterministic location models were developed for the commercial sector. They are characterized by fixed, known inputs such as demand levels, travel time and availability.

In the context of EMS, their primary shortcoming became very evident. Deterministic models assume a steady demand and resource availability. While it simplifies the mathematical formulation and computation, it fails to reflect real-world dynamics. While multiple authors suggested new adaptations to address certain elements, like incorporating backup strategies and "coverage tiers", they all attempt to simulate the idea of availability while being unable to do so in a structural manner.

The seven models discussed above represent only a very select overview of the extensive range of deterministic models. They were specifically chosen because they are relevant to the research question, show a development process that leads into the probabilistic models and are often used as stepping stones for newer models. For readers seeking a comprehensive understanding, in-depth literature reviews such as those by Farahani et al. provide detailed insights into the assumptions and applications of these models [22].

4 Probabilistic Models

Probabilistic models in Emergency Medical Services (EMS) location theory evolved as a response to the limitations of deterministic models, which assume fixed demand and guaranteed coverage. Deterministic approaches were originally developed for simplicity, often relying on manual or heuristic methods and making assumptions that ignored the stochastic nature of ambulance availability. However, with advancements in data availability and computational power, these simplifications became unnecessary.

Unlike deterministic models, probabilistic models embrace uncertainty by treating factors such as demand, travel time, and ambulance availability as random variables. They shift the focus from ensuring coverage under a fixed scenario to minimizing the risk of insufficient coverage. This paradigm shift better aligns with the unpredictable and dynamic nature of real-world EMS operations.

A central concept in probabilistic modeling is the **busy fraction**, the probability that an ambulance is occupied when a call arrives. This transforms coverage assessments from a binary framework (covered/not covered) into probabilistic expectations. Early models like the **Maximum Expected Covering Location Problem (MEXCLP)** introduced by Daskin (1983) incorporated this concept by assuming each ambulance has an equal, independent probability of being unavailable [23]. Later models, such as ReVelle and Hogan's **Maximum Availability Location Problem (MALP)**, refined this by allowing for uniform (MALP I) or location-specific (MALP II) busy probabilities, and optimizing for a desired coverage probability threshold (α) [24].

The rise of historical call data, GPS tracking, and real-time traffic information enables these models to better reflect local EMS patterns and demand variability. Enhanced algorithms and computing capabilities have significantly reduced solution times, even for large-scale problems. By using probabilistic models, policymakers can gain deeper insights into system performance, such as expected response times, the likelihood of simultaneous calls, and trade-offs between average and peak demand coverage. While these models introduce added complexity and sensitivity to input assumptions, they offer a powerful, data-driven framework for designing more adaptive and resilient EMS systems.

4.1 Maximum Expected Covering Location Model (MEXCLP)

The Maximum Expected Covering Location Model (MEXCLP) is an extension of the MCLP and first published by Daskin in 1983 [23]. While it is in no means the first probabilistic model that attempts to solve the probabilistic nature of EMS management, it is the first one to gain traction and became widely used. It does however use a similar approach to one that Chapman and White presented in 1974 called "Probabilitic Formulations of Emergency Service Facilities Location Problem" [25].

The MEXCLP makes some key assumptions:

- There exists a **predetermined fixed set of servers** that all operate independently of one another.
- All servers have the same **busy probability** q, regardless of their location or workload.
- There exists a **finite amount of demand points** that is either covered or not covered based on whether it lies within a fixed maximum response time or distance from a facility.
- The objective is solely to **maximize expected coverage** based on probabilistic availability.

4.1.1 Mathematical Formulation

Decision Variable

$$y_jk = \begin{cases} 1, \text{if node } k \text{ is covered by at least } j \text{ facilities} \\ 0, \text{if node } k \text{ is covered by less than } j \text{ facilities} \end{cases}$$

$$x_j = \text{The number of facilities (e.g., ambulances) located at site } j$$

Sets and Parameters

- V = Set of demand points.
- W = Set of candidate facility locations.
- W_i = Set of facility locations within range of demand point i.
- d_i = Demand weight at demand point i
- P = The fixed number of facilities available.
- q = Probability that a facility is unavailable (busy).
- A = Total number of available facilities.

Objective Function

$$\textbf{Maximize} \sum_{k=1}^{N} \sum_{j=1}^{M} d_i (1-q) q^{k-1} y_{ik} \tag{1}$$

The MEXCLP aims to maximize the expected covered demand calculated by summing the marginal contributions of each facility to each demand point.

Coverage Constraints

$$\sum_{j \in W_i} x_j \geq \sum_{k=1}^{p} y_{ik} \tag{2}$$

This ensures that the number of facilities within range of demand point i is at least equal to the number of coverage levels assigned to i.

Facility Constraints

$$\sum_{j \in W} x_j \leq A \tag{3}$$

This restricts the total number of facilities to be less than or equal to the available number.

Binary Constraints

$$y_{ik} \in 0, 1, \quad \forall i \in V, k = 1, ..., p \tag{4}$$

This ensures that the coverage variables are binary.

Integer constraints

$$x_j \in \mathbb{N}, \quad \forall j \in W \tag{5}$$

This ensures that the number of facilities at each location is a non-negative integer.

4.1.2 Example application in EMS

Consider a region with **five demand points**, **three candidate locations** and **two units** to be placed. A maximum coverage level of 2 ($P = 2$) which means that only 2 units can be placed on a location and a 30% chance that a facility is unavailable ($q = 0.3$). To solve this problem using MEXCLP, one could start by creating a travel time matrix (or distance matrix, depending on the set-up) $A = a_{ij}$, where i represents each intervention, j represents each ambulance and a_{ij} equals the travel time between the intervention site and the station.

$$A = \begin{bmatrix} 11 & 4 & 15 & 17 & 21 \\ 10 & 16 & 6 & 10 & 13 \\ 16 & 8 & 9 & 16 & 6 \end{bmatrix}$$

Using the aforementioned cutoff time of 12 minutes, a binary accessibility matrix B can be created where $b_{ij} = 1, \text{if } a_{ij} \leq 12$ or $b_{ij} = 0, \text{if } a_{ij} > 12$:

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Now consider the following demands d_i as presented in table below.

Demand point i	**Demand** d_i	**Candidate facility locations covering** i
1	100	$\{A, B\}$
2	80	$\{A, C\}$
3	120	$\{B, C\}$
4	60	$\{B\}$
5	90	$\{C\}$

Table 4.1: MEXCLP Demand and facility coverage table

Using the code provided in Appendix 4 - Example code MEXCLP, we can see that an ideal solution exists where units are located at facilities B and C, which covers all demand points once and demand point 3 twice.

4.1.3 Strengths and Limitations

The Maximum Expected Covering Location Problem (MEXCLP) is a powerful model for optimizing facility placement, particularly in emergency services, where probabilistic coverage is critical. One of its key strengths is its ability to account for the unavailability of facilities (e.g., ambulances) by incorporating a busy probability (q). This probabilistic approach provides a more realistic representation of coverage compared to deterministic models. Additionally, MEXCLP is computationally efficient due to its simplifying assumptions, such as uniform busy probabilities and deterministic coverage radii, making it suitable for practical applications in medium-to-large networks. The models focus on maximizing expected demand coverage ensures that high-demand areas receive priority, aligning well with real-world objectives.

However, MEXCLP has several limitations stemming from its simplifying assumptions. The assumption of independent servers and uniform busy probabilities across all locations may not reflect real-world conditions, where server availability is often correlated and location-dependent. The binary nature of coverage (either covered or not) ignores the gradual decline in service quality with increasing distance or response time. Furthermore, the model does not account for queueing effects during peak demand periods, which can significantly impact actual service levels. Another limitation is its fixed number of facilities, which restricts the model from determining the optimal number of facilities required for maximum coverage.

In summary, while MEXCLP is a robust and practical tool for facility location optimization under probabilistic conditions, its assumptions limit its realism in capturing complex system dynamics. Extensions like AMEXCLP and other advanced models address some of these shortcomings by relaxing certain assumptions and incorporating additional factors like queueing effects or location-specific busy probabilities. Nonetheless, MEXCLP remains a foundational model in location science and emergency service planning due to its balance between simplicity (and thus fast computation) and effectiveness.

4.2 Probabilistic Location Set Covering Problem (PLSCP)

The Probabilistic location set covering problem (PLSCP), first introduced by Chapman and White and later expanded by ReVelle and Hogan, uses local variability in busy fractions (proportion of time vehicles are responding to calls) in contrary to the previously discussed MEXCLP which uses uniform busy probabilities across all locations [25, 26]. The PLSCP makes some key assumptions:

- Accurately estimatable **sector-level busy fractions**
- A **probabilistic vehicle availability** that assumes that vehicle availability events are independent
- There exists a **finite amount of demand points** that is either covered or not covered based on whether it lies within a fixed maximum response time or distance from a facility.

4.2.1 Mathematical Formulation PLSCP

Decision Variable

$$x_j = \text{The number of facilities (e.g., ambulances) located at site } j$$

Sets and Parameters

- I = Set of demand nodes
- J = Set of candidate ambulance locations
- N_i = Set of facilities j that can cover demand point i within the maximum response time.
- f_k = the frequency of calls at demand node k (calls per day)
- t = the average durations of a service call in hours
- α = minimum acceptable probability that at least one vehicle within the coverage standard will be available to respond to a call.

Objective Function

$$\textbf{Minimize} Z = \sum_{j \in J} x_j \tag{1}$$

The PLSCP aims to minimize the total number of facilities.

Coverage Constraints

$$F_i = t \cdot \sum_{k \in M_i} f_k \tag{2}$$

This constraints describes the total daily hours of service that is required within the coverage zone around demand point i.

$$Q_i = \frac{F_i}{\sum_{J \in N_i} x_j} \tag{3}$$

$$1 - q_i^{\sum_{j \in N_i}} \geq \alpha \quad \forall i \in I \tag{4}$$

In equation 3, q_i represents the sector specific busy fraction for vehicles serving demand point i. Equation 4 ensures that the probability of having at least one available vehicle within the coverage standard meets the minimum reliability level.

$$1 - (\frac{F_i}{\sum_{j \in N_i}})^{\sum_{j \in N_i}} \geq \alpha \quad \forall i \in I \tag{5}$$

This coverage constraint is obtained by substituting the expression for q_i in equation 4. Since this constraint lacks an analytical solution, the model employs a numerical approach where the number of vehicles in N_i must equal or exceed the smallest integer b_i that satisfies:

$$1 - (\frac{F_i}{b_i})^{b_i} \geq \alpha \tag{6}$$

This results in the following lineair equivalent coverage constraint:

$$\sum_{j \in N_i} x_j \geq b_i \quad \forall i \in I \tag{7}$$

Binary Constraints

$$x_j \geq 0 \textbf{ and integer} \quad \forall j \in J \tag{8}$$

This constraint enforces that negative vehicle assignments are not permitted, fractional vehicle assignments are avoided and multiple assets can be placed on the same location.

4.2.2 Example application in EMS

Consider an urban region with six demand points and three candidate ambulance stations. The goal is to determine the minimum number of stations required so that 90% reliability is achieved for covering demand points within 12 minutes, considering an average service duration of t=1t=1 hour and the following daily call frequencies. To solve this problem,

Demand point	1	2	3	4	5	6
Calls/day (f_k)	4	3	5	2	6	1

one could start by creating a travel time matrix $A = a_{ij}$ where i represents each intervention, j represents each ambulance service and a_{ij} equals the travel time between the intervention site and the station.

$$A = \begin{bmatrix} 11 & 4 & 15 & 17 & 21 & 16 \\ 10 & 16 & 6 & 10 & 2 & 12 \\ 16 & 8 & 9 & 11 & 12 & 18 \end{bmatrix}$$

Using the aforementioned cutoff time of 12 minutes, a binary accessibility matrix B can be created where $b_{ij} = 1$, if $a_{ij} \leq 12$ or $b_{ij} = 0$, if $a_{ij} > 12$:

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Demand point i	**Candidate facility locations covering i**
1	$\{A, B\}$
2	$\{A, C\}$
3	$\{B, C\}$
4	$\{B, C\}$
5	$\{B\}$
6	$\{B\}$

Table 4.2: PLSCP facility coverage table.

Then we calculate for each demand point i, the demand intensity (F_i) using the formula $F_i = t \cdot \sum_{k \in M_i} f_k$ with M_i representing the demand points covered by i.

Demand point i	**Covered points M_i**	**F_i calculation**	**F_i value**
1	1,2	1*(4+3)	7
2	1,2	1*(4+3)	7
3	3,4	1*(5+2)	7
4	3,4	1*(5+2)	7
5	5	1*6	6
6	6	1*1	1

Table 4.3: PLSCP demand intensity table.

In the last step, we determine the minimum amount of vehicles required by solving $1 - (\frac{F_i}{b_i})^{b_i} \geq 0.9$for each demand point:

Demand point i	F_i **value**	Minimum b_i	Comment
1	7	10	
2	7	10	
3	7	10	
4	7	10	
5	6	9	
6	1	2	for $\alpha = 0.75$

Table 4.4: PLSCP demand intensity table.

By using an integer solving program with the following objective: $Minimize \quad Z = x_A + x_B + x_C$ we can determine that an optimal solution exists with station A, B an C having 9, 9 and 1 vehicles respectively.

4.2.3 Strengths and Limitations

The probabilistic location set covering problem (PLSCP) has some large advantages over older models like the LSCP. It does take the probabilistic nature of availabilities into account and can even account for regional differences. The models focus on minimizing the total amount of EMS assets by placing the in optimal location. Another strength of this model is that it allows the policymakers to calculate different availability probabilities and therefor can accurately calculate the price of an increase or decrease.

However, the PLSCP has a large dependency on the busy fraction, the estimate of how often assets are in use and thus unavailable. This factor is notoriously hard to accurately predict and even small differences can create a big difference in the amount of assets required to provide coverage. The LSCP also assumes that all assets operate independently, however in the Belgian context, multiple tiers of assets can be sent to the same incident, something this model inherently fails to simulate.

4.3 Maximum Availability Location Problem (MALP I & MALP II)

The Maximum Availability Location Problem (MALP) models, developed by ReVelle and Hogan (1989), were designed to improve ambulance location planning by considering the probability that ambulances may be busy when a call arrives [24]. These models aim to ensure that emergency demand points have a high probability of being served by an available ambulance within a specified response time.

The MALP-models make some key assumptions:

- Emergency calls follow a Poisson process.
- Ambulance busy probabilities are independent.
- All ambulances have similar service times.
- Demand patterns are assumed to be static during the planning period.

4.3.1 Mathematical Formulation MALP I

Decision Variable

$$x_i = \begin{cases} 1, & \textbf{if at location } i \textbf{ an ambulance is placed} \\ 0, & \textbf{if at location } i \textbf{ no ambulance is placed} \end{cases}$$

$$y_j = \begin{cases} 1, \textbf{if demand node } j \textbf{ is considered covered} \\ 0, \textbf{if demand node } j \textbf{ is not considered covered} \end{cases}$$

Sets and Parameters

- J = Set of demand nodes
- I = Set of candidate ambulance locations
- t_{ij} = Travel time from location i to node j
- τ = Maximum allowed response time
- a_j = Demand at node j
- q = Probability that an ambulance is busy
- B = Number of ambulances that are available
- α = Minimum acceptable availability probability

Objective Function

$$\textbf{Maximize} \sum_{j \in J} a_j y_j \tag{1}$$

The MALP I aims to maximize the total demand that is reliably covered.

Coverage Constraints

$$\sum_{i \in N_j} x_i \geq k \quad \forall j \in J \tag{2}$$

A demand node j is considered reliably covered if at least k ambulances can reach it withing time τ.

$$1 - q^{\sum_{i \in N_j} x_i} \geq \alpha \quad \forall j \in J \tag{3}$$

The probability that at least one ambulance is available for node j must be at least α.

Facility Constraints

$$\sum_{i \in I} x_i \leq B \tag{4}$$

This restricts the total number of facilities to be less than or equal to the available number.

Binary Constraints

$$x_i, y_j \in \{0, 1\} \quad \forall i \in I, j \in J \tag{5}$$

This ensures that the coverage variables are binary.

4.3.2 Strengths and Limitations of MALP I

Strengths:

- Considers ambulance unavailability due to being busy.
- Provides a probabilistic guarantee of coverage.
- Simple to implement with integer programming.

Limitations:

- Assumes all ambulances have the same busy probability.
- Does not consider queueing or multiple simultaneous calls at the same location.
- Assumes static demand and service rates.

4.3.3 Example application in EMS using MALP I

Consider a region with **six demand nodes**, **six potential ambulance locations**, and **three ambulances** to be placed. The goal is to **maximize the total demand** that is reliably covered using these three ambulances. To solve this problem, one could start by creating a travel matrix $A = a_{ij}$, where i represents each intervention, j represents each equipment location and a_{ij} represents the travel time between the intervention site and the station.

$$A = \begin{bmatrix} 11 & 5 & 18 & 16 & 13 & 14 \\ 2 & 8 & 13 & 15 & 19 & 31 \\ 15 & 3 & 5 & 11 & 16 & 18 \\ 18 & 16 & 11 & 6 & 10 & 16 \\ 21 & 13 & 18 & 7 & 4 & 11 \\ 15 & 14 & 18 & 19 & 3 & 1 \end{bmatrix}$$

Using a cutoff time of 12 minutes, a binary accessibility matrix B can be created where $b_{ij} = 1$ if $a_{ij} \leq 12$.

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Now consider a general unavailability probability $q = 0.3$, a minimum availability probability $\alpha = 0.7$ and the following population weights d_i as presented in the table below:

Demand point i	**Demand d_i**	**Candidate facility locations covering i**
1	100	$\{A, B\}$
2	150	$\{A, B, C\}$
3	120	$\{B, C, D\}$
4	130	$\{C, D, E\}$
5	110	$\{D, E, F\}$
6	140	E, F

Table 4.5: MALP I Demand and facility coverage table

Using the code provided in Appendix 5 - Example code PLSCP, we can see that an ideal solution exists where units are located at facilities A, B and E, which covers all demand points.

4.3.4 Mathematical Formulation MALP II

Decision Variable

$$x_i = \begin{cases} 1, & \textbf{if at location } i \textbf{ an ambulance is placed} \\ 0, & \textbf{if at location } i \textbf{ no ambulance is placed} \end{cases}$$

$$y_j = \begin{cases} 1, \textbf{if demand node } j \textbf{ is considered covered} \\ 0, \textbf{if demand node } j \textbf{ is not considered covered} \end{cases}$$

$$z_{ij} = \begin{cases} 1, \textbf{if ambulance on location } i \textbf{ is assigned to cover demand node } j \\ 0, \textbf{if ambulance on location } i \textbf{ is not assigned to cover demand node } j \end{cases}$$

Sets and Parameters

- J = Set of demand nodes
- I = Set of candidate ambulance locations
- t_{ij} = Travel time from location i to node j
- τ = Maximum allowed response time
- a_j = Demand at node j
- q = Probability that an ambulance is busy
- B = Number of ambulances that are available
- α = Minimum acceptable availability probability
- q_i = Probability that base i is busy

Objective Function

$$\textbf{Maximize} \sum_{j \in J} a_j (1 - \prod_{i \in N_j} (1 - (1 - q_i) Z_{ij}) \tag{1}$$

The MALP II aims to maximize the weighted coverage with reliability.

Coverage Constraints

$$\sum_{i \in N_j} x_i \geq k \quad \forall j \in J \tag{2}$$

A demand node j is considered reliably covered if at least k ambulances can reach it withing time τ.

$$1 - q^{\sum_{i \in N_j} x_i} \geq \alpha \quad \forall j \in J \tag{3}$$

The probability that at least one ambulance is available for node j must be at least α.

Facility Constraints

$$\sum_{i \in I} x_i \leq B \tag{4}$$

This restricts the total number of facilities to be less than or equal to the available number.

Binary Constraints

$$x_i, y_j, z_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J \tag{5}$$

This ensures that the coverage variables are binary.

Other Constraints

$$z_{ij} \leq x_i \quad \forall i \in I, j \in J \tag{6}$$

This constraints prevents coverage from unstaffed bases.

4.3.5 Strengths and Limitations of MALP II

Strengths:

- Accommodates varied busy probabilities across stations
- Enables differentiated reliability targets per demand zone
- More realistic representation of operational conditions

Limitations:

- Nonlinear formulation increases computational complexity
- Requires precise estimation of station-specific unavailability probabilities
- Still assumes independent ambulance availability

4.3.6 Example application in EMS using MALP II

Consider a region with **six demand nodes**, **six potential ambulance locations**, and **three ambulances** to be placed. The goal is to maximize the total demand that is reliably covered. To solve this problem, one could start by creating a travel matrix $A = a_{ij}$, where i represents each intervention, j represents each equipment location and a_{ij} represents the travel time between the intervention site and the station.

$$A = \begin{bmatrix} 11 & 5 & 18 & 16 & 13 & 14 \\ 2 & 8 & 13 & 15 & 19 & 31 \\ 15 & 3 & 5 & 11 & 16 & 18 \\ 18 & 16 & 11 & 6 & 10 & 16 \\ 21 & 13 & 18 & 7 & 4 & 11 \\ 15 & 14 & 18 & 19 & 3 & 1 \end{bmatrix}$$

Using a cutoff time of 12 minutes, a binary accessibility matrix B can be created where $b_{ij} = 1$ if $a_{ij} \leq 12$.

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Now consider a minimum availability probability $\alpha = 0.7$, the following population weights d_i and specific unavailability probabilities q as presented in the table below:

Demand point i	**Demand** d_i	**Candidate facility locations covering** i	**Specific unavailability probability**
1	100	$\{A, B\}$	0.25
2	150	$\{A, B, C\}$	0.3
3	120	$\{B, C, D\}$	0.35
4	130	$\{C, D, E\}$	0.2
5	110	$\{D, E, F\}$	0.4
6	140	E, F	0.15

Table 4.6: MALP II Demand, facility coverage and unavailability table

Using the code provided in Appendix 6 - Example code MALP I, we can see that an ideal solution exists where units are located at facilities A, D and E, which covers all demand points.

Please note that the PuLP-library that was used doesn't handle probabilistic (non-linear) problems, but is strictly designed for linear problems. While the dot product ($\sum q_i[i] * z[i, j]$) proivides a linear approximation of the probability coverage. While this maintains compatibility with PuLP solvers, it sacrifices accuracy and does not guarantee an optimal solution. For smaller datasets, the **NLopt** library can be used for nonlinear optimization. For larger datasets, **Pyomo** is more suitable, though both require advanced knowledge of nonlinear programming and significantly longer computation times.

4.4 Probabilistic Facility-Location, Equipment-Emplacement Technique (PROFLEET)

The Probabilistic Facility-Location, Equipment-Emplacement Technique (PROFLEET) model is probabilistic extension to the FLEET model, designed for determining the optimal placement of fire protection services, but can also be used for EMS [27]. This extension focuses specifically on using a probabilistic approach to model the unavailability of these assets. Like the FLEET model, it is capable of handling a multi-tier approach and it makes some key assumptions:

- There exists a static demand pattern represented by a **fixed set of demand points**.
- There exists **fixed set of candidate locations for primary resources**
- There exists **fixed set of candidate locations for specialized resources**
- There exists a **predefined maximum response time** (or in absence, a distance threshold) that determines coverage feasibility for both primary and specialized resources.
- The objectives are **binary coverage**; a demand point is either covered within the predefined maximum response time or not, there does not exist a partial coverage.
- There exists **a fixed amount of primary and specialized equipment** that need to be allocated.

4.4.1 Mathematical Formulation

Decision Variable

$$x_j^p = \begin{cases} 1, \text{if, or how many, primary equipment is located at site } j \\ 0, \text{otherwise} \end{cases}$$

$$x_j^s = \begin{cases} 1, \text{if, or how many, specialized equipment is located at site } j \\ 0, \text{otherwise} \end{cases}$$

$$z_j = \begin{cases} 1, \text{if an asset is located at candidate location } j \\ 0, \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, \textbf{if a demand node } i \textbf{ is covered by both primary and specialized equipment within their} \\ \textbf{respective standards and within the required reliability} \end{cases}$$

Sets and Parameters

- I = Set of demand points (indexed by i)).
- J = Set of candidate facility locations (indexed by j).
- NS_i = Set of stations within primary response time S^p of demand node i.
- Np_i = Set of stations within specialized equipment time S^s of demand node i.
- a_i = Demand at demand node i.
- p^p = The maximum number of primary equipment units to be located.
- p^s = The maximum number of specialized equipment units to be located.
- p^z = The maximum number of stations.
- α = The minimum reliability for both primary and specialized equipment.
- q_i^p = Local busy fraction for primary equipment in the coverage area of node i.
- q_i^s = Local busy fraction for specialized equipment in the coverage area of node i.

Objective Function

$$\textbf{Maximize} \quad \sum_{i \in I} a_i y_i \tag{1}$$

The PROTEAM aims to maximize the total covered demand within the required reliability α.

Coverage Constraints

$$1 - (q_i^p)^{\sum_{j \in Np_i} x_j^p} \geq \alpha \tag{2}$$

This constraint determines that at least one primary equipment is available within the minimum reliability. As this is equivalent to requiring that the number of primary equipment in the coverage area of node i is at least b_i^p, where b_i^p is the mallest integer satisfying following constraint:

$$1 - (q_i^p)^{b_i^p} \geq \alpha \tag{3}$$

Written as a linear constraint, we can thus write:

$$\sum_{j \in Np_i} x_j^p \geq b_i^p y_i \tag{4}$$

Similarly, for the specialized equipment, this constraint becomes:

$$\sum_{j \in Ns_i} x_j^s \geq b_i^s y_i \tag{5}$$

Resource Constraints

$$\sum_{j \in J} z_j \leq p^z \tag{6}$$

$$\sum_{j \in J} z_j^p \leq p^p \tag{7}$$

$$\sum_{j \in J} z_j^s \leq p^s \tag{8}$$

These constraints require that the model will not attempt to place more assets than is allowed.

$$x_j^p \leq z_j \tag{9}$$

$$x_j^s \leq z_j \tag{10}$$

These constraints require that both primary and specialized equipment can only be places once a station has been built.

Binary Constraints

$$z_j \in \{0,1\}, \quad \forall j \in J \tag{11}$$

$$y_i \in \{0,1\}, \quad \forall i \in I \tag{12}$$

$$x_j^p, x_j^s \in \{0,1\} \textbf{(or non-negative integer, depending on model variant)} \tag{13}$$

4.4.2 Example application in EMS

Consider an area with **six demand points** and **four candidate locations** for emergency facilities. The goal is to **maximize the total population** reliably covered within a response time of 12 minutes for primary equipment and 15 minutes for specialized equipment using up to three primary units and two specialized units, with no more than three stations opened. To solve this problem, one could start by creating a travel matrix $A^p = a_{ij}^p$ where i represents each demand location, j represents each equipement location and $A^s = a_{ij}^p$ equals the travel time between the demand locations and the station for primary equipment. Similarly, a second travel matrix $A^s = a_{ij}^s$ can be constructed for the specialized equipment.

$$A^p = \begin{bmatrix} 6 & 12 & 13 & 2 & 4 & 9 \\ 1 & 17 & 21 & 11 & 9 & 7 \\ 10 & 6 & 16 & 14 & 3 & 12 \\ 14 & 12 & 18 & 10 & 2 & 11 \end{bmatrix}$$

$$A^s = \begin{bmatrix} 11 & 4 & 15 & 17 & 21 & 16 \\ 10 & 16 & 6 & 10 & 2 & 12 \\ 16 & 8 & 9 & 11 & 12 & 18 \\ 12 & 10 & 14 & 9 & 11 & 15 \end{bmatrix}$$

Using the aforementioned cutoff time of 12 minutes for primary units and 15 minutes for specialized equipment, two binary accessibility matrices can be created:

$$B^p = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} B^s = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Demand point i	**Population**	**Primary Sites**	**Specialized Sites**
1	9,000	$\{A, B, C\}$	$\{A, B, D\}$
2	6,000	$\{A, C, D\}$	$\{A, C, D\}$
3	12,000	$\{A, D\}$	$\{A, B, C, D\}$
4	14,000	$\{A, B, D\}$	$\{B, C, D\}$
5	11,000	$\{A, B, C, D\}$	$\{B, C, D\}$
6	8,000	A, B, C, D	$\{B, D\}$

Table 4.7: PROFLEET Population and facility coverage

Using the code provided in Appendix 8 - Example code PROFLEET, the assumed busy fractions $q^p = 0.3$ **&** $p^s = 0.4$ and a required minimum availability ($\alpha = 0.7$), we can see that an ideal probabilistic coverage solution exists where primary units are located at facilities A, C and D, specialized units at facilities C and D.

4.4.3 Strenghts and Limitations

This enhanced probabilistic FLEET model presents several advantages for use in EMS. Firstly, it maintains its ability to model a multi-tier approach, common in many EMS systems. Secondly it provides a more realistic and operationally meaningful coverage model by incorporating probabilistic elements such as the local busy fractions.

Computational complexity becomes its main limitation, as is common with most probabilistic models. Large networks and areas can make the model to intensive to calculate at reasonable speeds. It also relies heavy on a detailed and accurate prediction of demand and local busy fractions, which may not always be feasible. Lastly, like all other discussed models, it fails to model temporal fluctuations which are also very common in EMS and might significantly impact the real-life performance of a result.

4.5 Hypercube Queuing Model (HQM)

The Hypercube Queuing Model (HQM) was developed by Larson in 1974 and uses a different method to simulate the EMS [28]. By modeling server availability states within an N-dimension hypercube structure, it is able to evaluate complex dispatching policies and workload distribution patterns. The HQM makes some key assumptions:

- **Demand follows Poisson process** at discrete atoms (geographical subunits).
- Service times follow **exponential distribution**.
- Servers are **distinguishable and mobile**.
- A **fixed-preference dispatching policy** (ordered backup list for each zone).
- System states represented as hypercube vertices where 0 represents available and 1 represents busy.
- **Steady-state balance equations** govern state transitions

4.5.1 Mathematical Formulation HQM

Decision Variable

$$x_j \in \{0,1\} : 1 \textbf{ if server placed as location } j$$
$$y_i \in \{0,1\} : 1 \textbf{ if demand node } i \textbf{ is covered}$$

Sets and Parameters

- I = Set of demand nodes
- J = Set of candidate ambulance locations
- a_i = the demand at node i
- q = The server busy probability
- α = The minimum required availability
- N_i = Set of facilities j that can cover demand point i

Objective Function

$$\textbf{Maximize} \sum_{i \in N_i} x_j \leq a_i y_i \tag{1}$$

The HQM aims to maximize coverage on the population level.

Coverage constraints

$$\sum_{j \in N_i} x_j \leq m y_i \quad \forall i \in I \tag{2}$$

The node i is considered covered if more than m servers exist in the set N_i, with:

$$m = \lceil \frac{\ln(1-\alpha)}{\ln q} \rceil = 2 \tag{3}$$

Facility Constraints

$$\sum_{j \in J} x_j = B \tag{4}$$

This constraint ensures that B assets are placed.

Binary Constraints

$$x_j, y_i \in \{0,1\} \quad \forall i \in I, j \in J \tag{5}$$

4.5.2 Example application in EMS

Consider an area with **six demand points** and **three candidate locations**. To solve this problem, the HQM starts by creating a travel matrix $A = a_{ij}$ where i represents each demand location, j represents each asset location and a_{ij} equals the travel time between the intervention site and the station.

$$A = \begin{bmatrix} 11 & 4 & 15 & 17 & 4 & 16 \\ 10 & 16 & 6 & 10 & 13 & 18 \\ 16 & 8 & 9 & 11 & 12 & 8 \end{bmatrix}$$

Using the aforementioned cutoff time of 12 minutes, a binary accessibility matrix B can be created where $b_{ij} = 1, \text{if } a_{ij} \leq 12$ or $b_{ij} = 0, \text{if } a_{ij} > 12$:

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Demand point i	Population	**Candidate facility locations covering i**
1	100	$\{A, B\}$
2	150	$\{A, C\}$
3	120	$\{B, C\}$
4	130	$\{B, C\}$
5	110	$\{A, C\}$
6	140	$\{C\}$

Table 4.8: HQM facility coverage table.

Using the code provided in Appendix 9 - Example code HQM, we can see that an optimal solution exists where 2 units are located at facilities A and C, and thus a maximal demand with a population of 260 can be covered.

4.5.3 Strengths and Limitations

The Hypercube Queuing Model (HQM) has advantages over the previously described models as it is the first one to use a multi-dimensional hypercube to model server interdependence as present in EMS. It enables precise workload imbalance analysis by supporting complex dispatching hierarchies.

As the status of each node is modeled, every time the model gets updated (the computational complexity), the HQM experiences limited scalability ($O(2^N)$. It also requires some assumptions for the steady-state and parameter estimations, which can be very difficult to accurately estimate for real-world systems.

4.5.4 Strengths and Limitations

Probabilistic models represent a significant advancement over deterministic approaches by acknowledging the inherent uncertainty in EMS operations. Their primary strength lies in their ability to model ambulance availability more realistically, providing expected coverage calculations rather than binary assessments. Models like **MEXCLP** can account for backup coverage, where multiple ambulances can serve the same demand point, significantly improving system reliability.

The integration of queueing theory, particularly through models like the **hypercube queuing models (HQM)**, allows for sophisticated analysis of server interactions and spatial variations in workload. These models can capture the dependency between ambulances and provide detailed performance measures including response time distributions, utilization rates, and coverage probabilities.

However, probabilistic models face several critical limitations. The most significant challenge is the accurate determination of busy fractions, which should theoretically depend on the number and distribution of ambulances between stations, yet are often treated as exogenous inputs. This creates a circular dependency problem where the busy fraction is both an input to and output of the location model. Iterative approaches have been proposed to address this, but they provide heuristic rather than globally optimal solutions.

The assumption of independence between ambulance availabilities, while computationally convenient, often fails to reflect real-world interdependencies where ambulances may be correlated in their busy states due to incident clustering or system-wide demand surges.

Furthermore, most probabilistic models assume stationary demand patterns and use average arrival rates, whereas actual EMS call data shows significant temporal variations throughout the day, week, and season. The models also typically ignore operational constraints such as crew schedules, vehicle maintenance, and regulatory requirements that significantly impact real-world availability.

5 Dynamic Models

5.1 Introduction

To address the temporal variability of demand, resource availability, and environmental constraints, Emergency Medical Services (EMS) location models have increasingly adopted dynamic approaches. While traditional static and probabilistic models offer foundational tools for planning, they often fall short in accounting for real-time variability and operational volatility. Dynamic models, by contrast, incorporate time-dependent decision-making capabilities, enabling EMS systems to adapt proactively to changing conditions. These models align more closely with real-world complexities, where demand can surge unexpectedly, resources are mobile, and traffic patterns fluctuate throughout the day.

This chapter explores the evolution of dynamic EMS location models, highlighting their primary strengths and limitations. It also presents four research-informed methodologies that illustrate the range of dynamic modeling strategies currently in use.

5.2 Strengths of Dynamic EMS Location Models

Dynamic models are especially effective in environments where EMS demand shifts hourly, daily, or seasonally. For instance, urban systems frequently experience peak demand during rush hours, weekends, or public events. By incorporating time-series data, dynamic models can anticipate these variations and proactively adjust ambulance deployment strategies. This adaptability ensures higher service levels during peak times while avoiding inefficiencies during off-peak periods.

Static models tend to over-resource during low-demand intervals or under-resource during peak times. Dynamic approaches address this by continuously relocating idle ambulances to areas with emerging risk. The Integer Programming Model by Moeini et al. (2014), for example, demonstrated a 12–18% reduction in average response times by optimizing ambulance placement in real time within simulated urban environments [29].

Advanced dynamic models increasingly rely on real-time data from IoT sensors, GPS tracking, and traffic monitoring systems. These data streams feed into decision algorithms that update ambulance routes and stationing in real time. The Multiobjective Variable Neighborhood Strategy Adaptive Search (M-VaNSAS) model exemplifies this capability by adjusting routes based on live traffic speeds, thereby ensuring timely responses even in congested areas [30].

Unlike probabilistic models that typically focus on demand uncertainty, dynamic frameworks can incorporate multiple layers of uncertainty—such as ambulance availability, road closures, staff shortages, and adverse weather conditions. This makes them more robust in real-world deployments, where unexpected disruptions can occur simultaneously and must be managed in an integrated fashion.

5.3 Weaknesses of Dynamic EMS Location Models

Dynamic models require frequent recalculations as they continuously adapt to new data inputs. This can lead to significant computational overhead, especially in large or dense urban networks. For example, the System Dynamics Model by Martin and Bacaksizlar (2017) requires hourly updates to ambulance availability and demand forecasts, necessitating access to high-performance computing infrastructure, which is often unavailable in smaller or rural EMS systems [31].

These models depend heavily on high-quality, real-time data. Incomplete or erroneous inputs can severely degrade model performance. In a 2022 case study of Jakarta's EMS network, gaps in traffic speed data—due to sensor malfunctions—led to a 22% reduction in coverage optimization effectiveness using M-VaNSAS [30]. Such vulnerabilities highlight the importance of data integrity in dynamic EMS modeling.

Frequent relocation of ambulances can strain staff and increase operating costs related to fuel consumption and vehicle wear. For instance, simulations of the Dynamic Floating Stations Model showed that while temporary bases improved weekend coverage by 15%, the resulting increase in daily station shifts added approximately $280,000 per year in operational costs per 100 vehicles [32].

Dynamic models optimized solely on historical data can become brittle when exposed to novel conditions. Overfitting may lead to suboptimal performance in unprecedented scenarios such as natural disasters or major infrastructure failures.

A 2019 validation of dynamic integer programming models revealed a 34% increase in response times during unexpected flood events when compared to hybrid models that incorporated both static and dynamic components [31, 33].

5.4 Exemplary Dynamic EMS Location Models

The Time-Staged Integer Programming model by Moeini et al. (2014) divides the day into discrete time blocks, such as two-hour periods, solving separate optimization problems for each interval to determine optimal ambulance locations [29]. In a city EMS system divided into 12 two-hour blocks, the model relocates 30% of ambulances to highway-adjacent stations during morning rush hours (7–9 AM). By noon, vehicles shift toward commercial districts, where pedestrian activity peaks, improving responsiveness to lunchtime incidents.

The Multiobjective Variable Neighborhood Strategy Adaptive Search (M-VaNSAS) uses metaheuristics and real-time IoT data to simultaneously minimize response time, maximize skilled staff utilization, and reduce relocation costs [30]. During a citywide marathon, real-time traffic sensors detect congestion near race routes. M-VaNSAS reroutes ambulances to less congested areas while ensuring that each vehicle is staffed with a paramedic with at least five years of experience, balancing both speed and quality of care.

System Dynamics Fleet Allocation, proposed by Martin and Bacaksizlar (2017), employs stock-and-flow modeling to simulate long-term relationships between population trends, ambulance fleet size, and maintenance cycles [31]. A county anticipates a 20% population increase over 15 years. The model recommends scaling up the ambulance fleet by adding six vehicles in Years 1–5 and eight more in Years 6–10, maintaining sub-eight-minute response targets without overshooting budget constraints.

The Dynamic Floating Stations model (TMU, 2020) establishes modular EMS bases during forecasted demand spikes, such as concerts, sports games, or festivals [32]. For a 50,000-person music festival, three floating stations, each staffed with two ambulances, are deployed near the event site. Post-event evaluation reveals a 40% reduction in response times compared to scenarios relying solely on fixed stations.

5.5 Barriers to Implementing Dynamic EMS Models in Belgium

Despite the demonstrated benefits of dynamic EMS location models in terms of responsiveness and adaptability, their implementation in Belgium remains highly unlikely in the near term. This is primarily due to structural, technological, and policy-related limitations that hinder the development of the necessary ecosystem for such systems to function effectively.

One of the most critical barriers is the absence of real-time tracking infrastructure for ambulances and other EMS assets. Although Belgium has made significant progress in terms of 5G network coverage, enabling the technical capacity for such systems, no meaningful investments have been made to equip ambulances with GPS tracking or telemetry systems. As a result, dispatchers lack the live operational data required to feed into a dynamic model, making responsive fleet reallocation infeasible.

Moreover, real-time data exchange with hospitals is currently nonexistent. Information about hospital capacities, emergency department congestion, or specialized service availability is not transmitted through continuous data streams. This lack of integration severely limits the possibility of tailoring ambulance deployment or routing based on downstream capacity constraints, a cornerstone of dynamic EMS systems.

In addition to data availability issues, Belgium also faces significant limitations in its technical infrastructure. The country lacks the low-latency processing capabilities and high server or cloud capacity necessary to handle and compute dynamic optimization algorithms in real time. Without robust computational resources, even the best-designed models would be too slow or unstable to support operational decision-making at scale.

Paradoxically, while many countries struggle with fragmented EMS governance, Belgium benefits from a centralized EMS oversight structure, managed by the FPS Public Health and FPS Public Affairs. In theory, this centralization could facilitate system-wide implementation of dynamic models. However, in practice, this potential is undermined by policy decisions that have reinforced a rigid and static operational model.

A notable example is the 2024 Royal Decree, which significantly increased public subsidies for the development of physical ambulance bases. These investments came with strict requirements regarding base infrastructure, location, and staffing. As a result, service providers have been incentivized to adopt fixed-position operations to meet funding criteria. This commitment to physical infrastructure, while beneficial in ensuring regional coverage, has unintentionally disincentivized mobile or dynamically adjusted deployment strategies.

Additionally, ambulance accreditation and positioning processes are highly regulated and offer limited flexibility, further reinforcing the static nature of Belgium's EMS model. Providers must demonstrate long-term operational stability and compliance with base-related standards, reducing their ability to experiment with or transition toward more fluid, real-time models of deployment.

Furthermore, the cultural orientation of Belgium's EMS system—favoring predictability, regulation, and uniformity—may also limit the appetite for adopting systems that require decentralized autonomy, real-time adjustments, and algorithm-driven decision-making.

In summary, while Belgium possesses some enabling conditions, such as advanced telecommunications infrastructure and centralized governance, the broader landscape is not conducive to the adoption of dynamic EMS location models. The absence of real-time tracking, insufficient hospital data integration, limited processing infrastructure, and policy-driven rigidity collectively create a system that favors stability over responsiveness. Without significant strategic shifts in both technology investment and regulatory philosophy, dynamic EMS modeling is unlikely to take root in Belgium in the foreseeable future. Therefor, these models will not be discussed further in this thesis.

6 Scenario-based EMS simulation

This chapter presents a simulation-based study aimed at evaluating the suitability of several established location theory models for the reallocation of EMS assets in Belgium during a mass casualty incident (MCI). As discussed in the preceding chapters, the Belgian EMS system is structured as a fixed-base deployment model, with assets (ambulances, PITs, and MUGs) stationed at legally defined locations to serve the baseline demand. During a crisis, some of these assets may be temporarily reallocated to support surge operations under the MIP (Medical Intervention Plan).

The core problem addressed here is how to identify an optimal or near-optimal subset of EMS assets that can be reallocated to surge operations without causing unacceptable disruption to baseline coverage. Rather than building a new model from scratch, this research investigates how existing facility-location models perform when applied to this constrained, high-stakes, and time-sensitive problem.

The simulation process consists of three main components:

1. **Modeling EMS demand** across Belgium using demographic and geographic proxies.
2. **Defining a simplified but representative network** to simulate the effects of reallocation.
3. **Applying five selected location models** to identify which assets can be shifted in different crisis scenarios.

The chapter begins by detailing the EMS-approach in Belgium.

6.1 The Belgium EMS System: From Baseline to Surge Capacity

To appreciate the value and constraints of EMS location models, it is first necessary to understand the structure and functioning of the Belgian emergency medical services (EMS) system. Rooted in historical developments and embedded in complex legal frameworks, the Belgian EMS aims to ensure equitable and effective care both in daily operations and during large-scale incidents. The system balances a robust baseline capacity with a flexible surge capability, allowing for scalability in response to mass casualty incidents (MCIs) or public health threats.

At the core of this system lies a three-tiered response model that organizes EMS assets based on medical urgency and resource specialization. This chapter begins with an overview of the baseline EMS system that operates under normal circumstances, then introduces the surge EMS framework that is activated in times of crisis, and finally, discusses the theoretical underpinnings of EMS demand in modeling contexts.

6.1.1 Baseline EMS in Belgium

Belgium's baseline EMS system is grounded in a well-established legal and operational framework. It was first formalized by the Law of 1964, which laid the foundations for public emergency medical assistance. Subsequent reforms, including the most recent Royal Decree of 2024, have modernized the system to adapt to medical, demographic, and technological developments.

At the operational level, the system employs a three-tiered structure, designed to align the level of medical expertise and equipment with the severity of each incident. This pyramid-like model enables scalable, efficient deployment:

- **Ambulance services (Ambu)**: This foundational tier consists of ambulances staffed by two trained ambulance technicians, capable of providing basic life support (BLS), initial patient assessment, and transport. These units respond to the majority of non-critical cases and ensure widespread coverage.
- **Paramedical Intervention Team (PIT)**: The second tier includes an emergency nurse and an ambulance technician, trained in advanced life support (ALS). PIT units can administer medication and perform more advanced procedures using standardized protocols. They typically cover larger populations than Ambu units.
- **Mobile Emergency and Resuscitation Service (MERS)**: The highest tier involves a team of an emergency physician and a specialized nurse, capable of advanced trauma life support and critical interventions on-site. These are dispatched to the most severe emergencies, such as cardiac arrest or major trauma, and can act as mobile extensions of emergency departments.

All units are dispatched through a decentralized coordination system, guided by the Belgian Health Medical Regulation (BHMR) protocols, which aim to align the right level of care with the patient's condition, minimizing both under- and over-utilization.

These EMS tiers are operated by a mix of public and private entities, all adhering to nationally defined standards and integrated under a unified regulatory system. Each unit functions from a permanent physical base, as mandated by the 2024 Royal Decree. While this ensures long-term infrastructural investment and stability, it also constrains operational flexibility, making dynamic or real-time relocation of EMS units less feasible. To mitigate the financial burden and support static deployments, the federal government guarantees long-term operational funding to all EMS providers.

In summary, Belgium's baseline EMS framework is legally robust, medically stratified, and geographically stable, characteristics that shape the operational boundaries of EMS planning and simulation.

6.1.2 Surge EMS in Belgium

In situations that exceed the capacity of the baseline EMS, such as mass casualty incidents (MCI), large-scale accidents, or public health emergencies, Belgium activates a layered surge response system, fully integrated into its broader civil protection and crisis management structure.

This structure operates under the national crisis management framework, which organizes emergency response into five operational disciplines, each governed by legal instruments including the Royal Decree of 26 April 2024. Each discipline has its own mono-disciplinary contingency plan.

For EMS, Discipline 2: Medical is the most relevant and encompasses all aspects of health-related emergency response. This discipline is subdivided into four specialized plans:

- **Medical Intervention Plan (MIP)**: Coordinates the deployment of EMS assets during an MCI, including triage, resource allocation, and on-site coordination.
- **Psychosocial Intervention Plan (PSIP)**: Provides mental health and social support for victims, witnesses, and families during and after the crisis.
- **Sanitary Intervention Plan (SIP)**: Addresses public health risks such as infectious diseases, contamination, and environmental hazards.
- **Plan for Risk Manifestations (PRIMA)**: Prepares for high-risk, high-density events such as festivals, demonstrations, or public gatherings where medical emergencies may be more likely.

This thesis focuses exclusively on the Medical Intervention Plan (MIP), as it directly governs the surge deployment of EMS assets in crisis situations and is most relevant for simulation modeling.

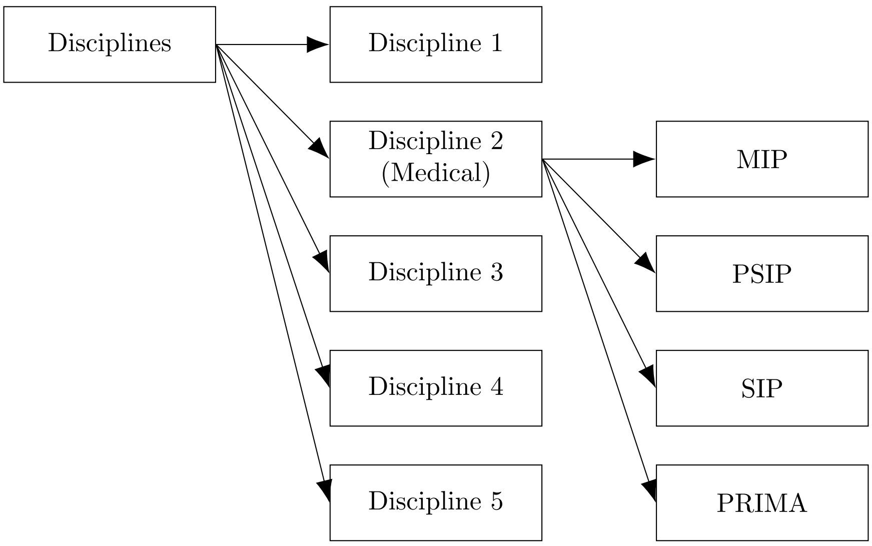


Figure 6.1: Overview of Belgian Crisis Response Structure: A crisis triggers five disciplines, with Discipline 2 (Medical) branching into four specialized intervention plans.

6.2 EMS Demand Modeling and Spatial Structure

6.2.1 Conceptualizing EMS Demand in the Belgian Context

Core Assumption: Population as the foundation of EMS Demand

In Belgium, population data is reported at the municipality level, where both geographic size and population density can vary dramatically. This variability complicates the use of raw population figures to estimate EMS demand. A more accurate measure is population density, which reflects how tightly inhabitants are concentrated and serves as a better proxy for potential EMS service needs.

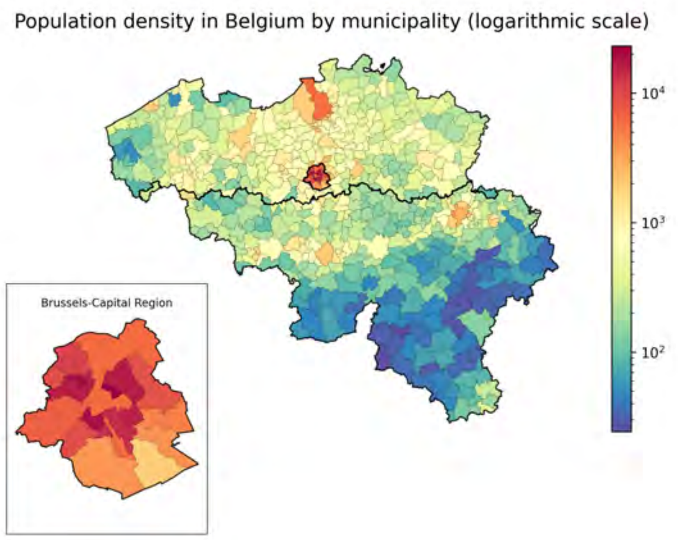


Figure 6.2: A map of Belgium representing the logarithmic population densities in its municipalities

These variations are substantial. For example, Saint-Josse-ten-Noode (Nl: Sint-Joost-ten-Node, Fr: Saint-Josse-ten-Noode) is Belgium's smallest municipality, covering just 1.16 km^2, but has the highest population density at 23,173 inhabitants per km^2. In contrast, Bastogne spans 264.69 km^2, making it nearly 230 times larger, while Vresse-sur-Semois has the lowest population density, with just 24 inhabitants per km^2.

To visualize these disparities, a logarithmic transformation was applied to population density values before mapping them, allowing both densely and sparsely populated municipalities to be meaningfully represented (Figure 6.2).

Given these extreme differences, it is essential to account for both size and density when implementing a location model. Whether for baseline EMS placement or surge reallocation, using population density instead of raw population helps avoid skewed results and supports more realistic modeling of demand.

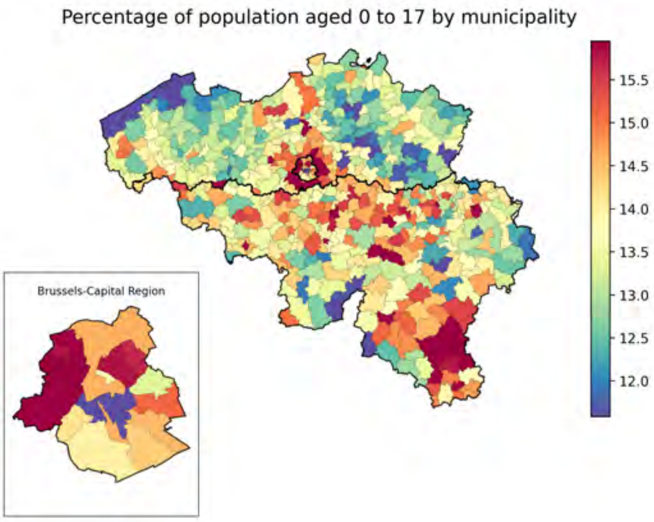
Beyond population density, demographic composition, particularly age structure, can also influence EMS demand. Research has shown that age is a significant factor in determining the likelihood of EMS utilization, either directly or through proxy data such as emergency department visits [34, 35, 36].

Although no national studies on age-based EMS demand exist for Belgium, demographic data reveal substantial variation in age distribution across municipalities. Figure 6.3 illustrates the percentage of the population under 18, between 18 and 65, and over 65 years old in each municipality. Yellow represents the national average, while blue and red show minimum and maximum extremes, respectively.

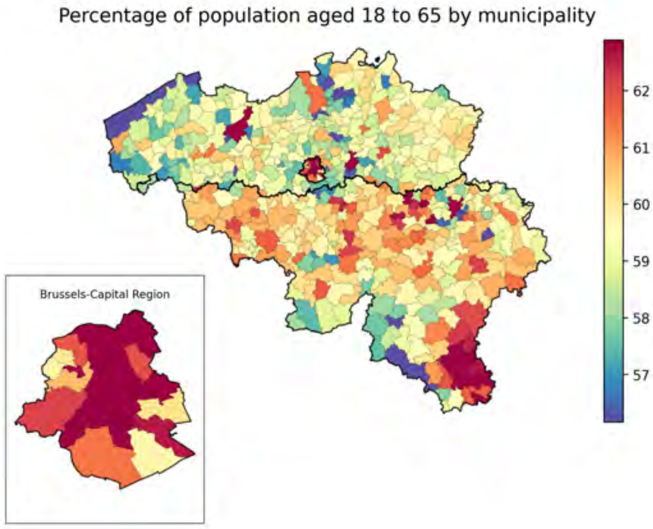
Studies indicate that EMS demand increases significantly among the elderly, particularly those over 85 years old. Conversely, pediatric calls are more common in adolescents than in younger children [36, 37, 38, 39]. These findings suggest that age-weighted models could provide a more nuanced understanding of EMS demand.

Figure 6.4 highlights this further by mapping the percentage of the population under six years old and over 85 years old, two high-utilization groups, by municipality. The data reveal notable geographic variation in the distribution of these age groups, suggesting that municipalities with higher concentrations of elderly or very young residents may experience elevated baseline EMS demand.

While this thesis does not explicitly incorporate age distribution in its simulation model, due to data constraints and complexity, the potential impact of demographic structure is acknowledged. Future modeling efforts may benefit from integrating age-stratified demand weights, especially in regions with aging populations or disproportionately high youth populations.



(a)



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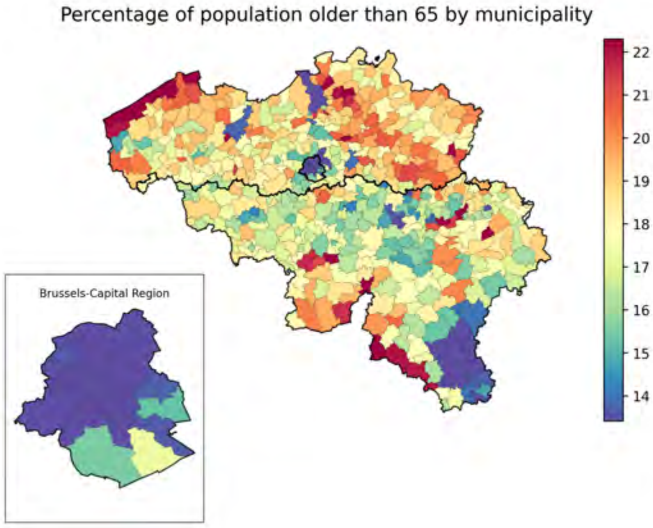


(c)

Figure 6.3: Percentage of population under 18 years old (a), between 18 and 65 years old (b), and older than 65 (c) by municipality

Temporal Variations in EMS Demand

In addition to spatial and demographic factors, temporal fluctuations in EMS demand represent a critical dimension when modeling or planning EMS resource allocation. These variations can occur by time of day, day of week, and even by season, and they have the potential to significantly impact the efficiency and responsiveness of EMS systems.

A systematic review by Cantwell et al. identified 38 studies that examined temporal patterns in EMS demand. Of these, 32 studies found clear evidence of time-based fluctuations [40]. The most commonly observed pattern across these studies was a bimodal distribution, with demand peaking once in the morning hours (typically between 08:00–09:00) and again in the early evening (around 18:00–19:00).

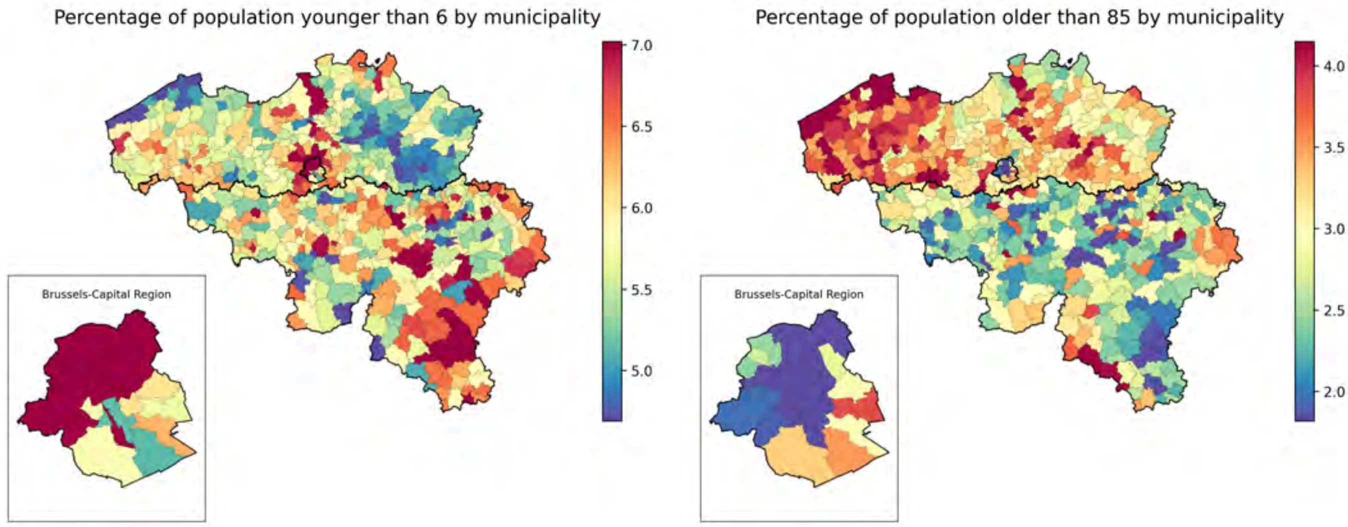
However, some studies failed to detect consistent temporal patterns. This inconsistency highlights the need for localized, data-driven assessments before implementing any model that accounts for time-of-day effects. Cantwell et al. also noted that demographic factors may act as confounders in temporal analyses, reinforcing the relevance of the age distribution variations discussed earlier.

To evaluate whether such temporal patterns are relevant in the Belgian context, this thesis examined EMS intervention data for the year 2024, encompassing all calls that led to dispatches. As shown in Figure 6.5, the Belgian data do reveal clear peaks in EMS demand, though at slightly different times than the international average. The Belgian peaks occur between 10:00–11:00 and again between 13:00–14:00, possibly reflecting local lifestyle or healthcare access patterns.

Further temporal trends are also evident:

During weekdays, EMS demand tends to decrease during the evening and night hours (17:00–06:00).

Conversely, during weekends, the evening hours see higher-than-average EMS demand, possibly due to changes in activity



(a) (b)

Figure 6.4: Percentage of population under 6 years old (a) and older than 85 years old (b) by municipality

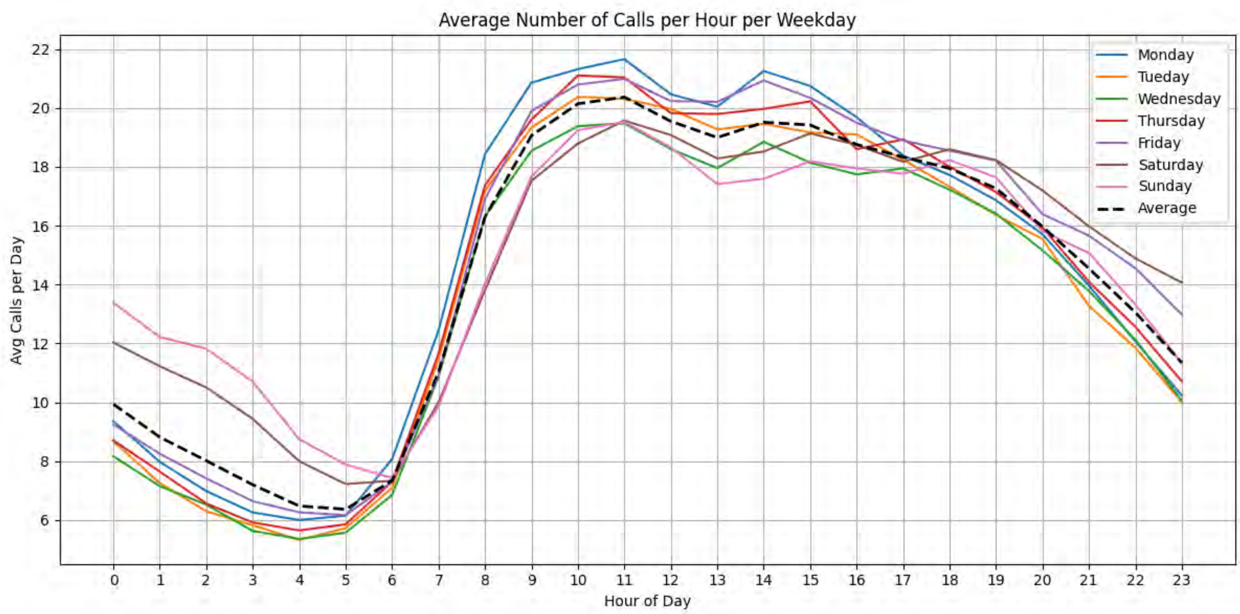


Figure 6.5: Temporal variations in Belgian EMS-calls in 2024

patterns, increased social events, or reduced access to other healthcare services.

While these observations do not provide conclusive proof of causality, they do support the idea that time-sensitive models could enhance EMS planning, particularly for dynamic or real-time dispatch systems. However, as with spatial and demographic refinements, further investigation is warranted before integrating temporal components into any operational EMS location model.

In this thesis, temporal variation is acknowledged as a relevant influence, but is not explicitly modeled in the simulation phase due to a focus on structural (rather than real-time) asset reallocation. Nonetheless, any future deployment or real-world application of location models in EMS should consider incorporating time-of-day and day-of-week demand variability to enhance responsiveness.

6.3 Simulation Setup

The goal of this simulation was to examine how various location theory models, identified through a structured scoping review, impact the selection of EMS assets for reallocation during major crisis scenarios in Belgium. While this simulation does not claim to definitively demonstrate improvements in service performance or outcomes, it aims to explore whether these models lead to systematically different allocation patterns and whether such changes have the potential to enhance population coverage and equity under surge conditions.

Due to the simplified nature of the demand data (municipality-level aggregates) and the hypothetical nature of the scenarios, results should be interpreted as indicative of potential system changes rather than conclusive performance evaluations.

6.3.1 Simulation Scope and Approach

The simulation models the spatial impact of activating EMS surge capacity during a Mass Casualty Incident (MCI), specifically through the temporary reallocation of ambulance assets to affected regions. No temporal dynamics, real-time queues, or feedback loops were incorporated; instead, the simulation focused on discrete system states following a reallocation "event."

Each simulation step involved:

- Selecting a group of ambulance assets for reallocation, as defined by a location theory model.
- Removing these assets from the baseline EMS pool.
- Measuring the population coverage of the remaining assets, defined as the number of people within a 15-minute road-based travel radius from any ambulance.
- Assessing the equity of coverage using Lorenz curves and the Gini coefficient.

This approach allows for comparison between different reallocation strategies in terms of coverage magnitude and distributional fairness, which are both critical in surge EMS planning.

6.3.2 Model selection

Based on the outcomes of the preceding scoping review, a total of twelve EMS location models were identified as candidates for simulation. Each was reviewed in terms of its theoretical structure, data requirements, and compatibility with the simulation framework used in this thesis.

Only three models were selected for implementation, based on their feasibility given the available data, their compatibility with a one-tier (ambulance-only) simulation, and their methodological comparability. The remaining models were excluded for reasons detailed below.

Models Included in the Simulation

- **Maximal Covering Location Problem (MCLP)**: This model seeks to maximize the population covered within a specified service radius given a fixed number of resources. It aligns well with the simulation's fixed-supply, static-demand setting.
- **Maximal Backup Coverage Problem (BACOP I & II)**: These models prioritize both primary and secondary (backup) coverage, aiming to ensure system redundancy. They were included due to their relevance in high-stress MCI scenarios, where multiple simultaneous demands may overwhelm primary coverage.
- **Maximum Expected Covering Location Model (MEXCLP)**: MEXCLP integrates probabilistic elements by estimating the likelihood that an ambulance is available at the time of call. In this simulation, its implementation was simplified by using standard assumptions about availability, enabling comparative evaluation.

Models Excluded from the Simulation

- **Location Set Covering Problem (LSCP)**: This model assumes that complete coverage of all demand nodes is required, which is incompatible with the surge scenario where a portion of the ambulance fleet is removed.
- **Tandem Equipment Allocation Model (TEAM)** and **Facility-Location, Equipment-Emplacement Technique (FLEET)**: Both are designed for multi-tiered EMS systems, such as those using ambulances, PIT units, and SMURs. As this simulation focused only on ambulances, these models were excluded.
- **Double Standard Model**: This model operates under a different optimization paradigm and could not be integrated into the comparative simulation framework.
- **Probabilistic Location Set Covering Problem (PLSCP)** and **Maximum Availability Location Problem (MALP I & II)**: These models require accurate estimates of asset availability/unavailability ratios. As no validated real-world data were available for this in the Belgian context, they were excluded.
- **Probabilistic FLEET (PROFLEET)**: Like FLEET, this model is multi-tiered and probabilistic, making it unsuitable for a single-tier, deterministic simulation.
- **Hypercube Queuing Model (HQM)**: The HQM uses a hypercube rather than a road network and is therefore not directly comparable to the network-based spatial coverage outputs used here.
- **All dynamic models**: Entire classes of dynamic or time-sensitive location models were excluded, as they require real-time data inputs and simulations of system evolution over time. Such data were not available or reproducible for this study.

6.3.3 Data Sources and Preprocessing

Ambulance Assets

The list of accredited ambulance assets and their base locations was provided by the FPS Public Health under the obligations of the 2024 Royal Decree, which mandates formal registration of all operational ambulances. This dataset included 467 distinct assets, each geolocated to a known operational base.

Population Demand

Municipality-level population data were obtained from the FPS Interior, using the most recent dataset available for 2024. Population totals were assigned to calculated geographical centroids of each of the 565 Belgian municipalities. Notably, these centroids were not based on administrative centers, as such locations are often historically skewed or spatially clustered due to non-geometric factors. Instead, true geographic centroids were used, offering a more uniform approximation of spatial demand.

To ensure compatibility with the road network used in the simulation, each centroid was snapped to the nearest road segment using a built-in OSRM function. This ensured that all demand points were accessible within the routing environment.

Travel Time Matrix and Network Setup

Routing and travel-time calculations were carried out using a local OSRM (Open Source Routing Machine) server configured with the Belgian OpenStreetMap dataset dated March 12, 2025. The routing engine was set to use the car profile, but was customized to ignore local speed limits, in line with the assumption that EMS vehicles operate with exemption from normal traffic restrictions during emergency response.

The OSRM setup accounted for:

- Real road conditions, including blocked streets, one-way systems and pedestrian-only zones.
- Ambulance station points as origins.
- Municipality centroids (snapped to roads) as destinations.

This produced a full ambulance-to-municipality accessibility matrix, used to compute coverage outcomes in each simulation step.

6.3.4 Scenario Design

To explore the variability in surge response needs, five hypothetical MCI scenarios were constructed. Each scenario involved a different scale of ambulance reallocation, corresponding to tiers of surge activation defined in Belgium's Medical Intervention Plan (MIP). An overview of how these scenarios were defined can be found in 6.1. These scenarios represent a range of realistic conditions in which MIPs would be activated. By simulating spatial reallocation within each case, the model explores the relative potential for improved accessibility, compared to unoptimized distributions.

Scenario	Context	MIP Type	Reallocated assets	Asset Pool
1	Low-density rural area	Standard	5 ambulances	Intra-provincial
2	Mixed-density area	Extended	20 ambulances	Intra-provincial and neighboring
3	Urban high-density city	Extended	20 ambulances	Intra-provincial and neighboring
4	Same as scenario 1	MAXI	40 ambulances	National
5	Same as scenario 3	MAXI	40 ambulances	National

Table 6.1: Overview of Simulation Scenarios

6.3.5 Simulation Outputs

Each simulation produced two core outputs:

- **Total Population Covered**: Defined as the number of residents located within 15 minutes' driving distance of an operational ambulance unit, based on OSRM travel-time calculations.
- **Equity of Coverage**:
  - **Lorenz Curve**: A visual representation of how coverage is distributed across municipalities.
  - **Gini Coefficient**: A scalar measure of inequality (0 = perfect equity, 1 = total inequity).

These outputs were calculated separately for each location model across all five scenarios. Together, they allowed for comparative analysis of not only how much of the population remains covered, but also how evenly that coverage is distributed.

6.4 Simulation Results

6.4.1 Scenario 1: Rural MIP Activation (5 Assets Reallocated - Standard MIP)

In the first scenario, a minimal activation of surge capacity was simulated, involving the reallocation of **five ambulance assets** across the network. Despite the small reduction in system resources, this scenario already demonstrated notable differences between model behaviors.

Coverage Outcomes

Most models successfully preserved full coverage of all demand locations within the affected province, with the exception of the *Skip-One heuristic* and the *BACOP I model*, both of which failed to maintain complete coverage. This result is particularly notable for BACOP I, which is designed to preserve redundancy and robustness in coverage. The limited scale of reallocation suggests that even minor adjustments can disrupt sensitive configurations when not properly balanced.

Equity Outcomes

Evaluation of equity using the *Lorenz curve* and *Gini coefficient* revealed a somewhat surprising trend. Across all models, except Skip-One, the Gini coefficient increased relative to the baseline configuration, indicating a reduction in equity. However, the Skip-One heuristic produced a **lower Gini coefficient than the baseline**, suggesting a more even distribution of coverage, despite its failure to achieve complete geographic coverage.

Among all tested models, the *2-on, 1-off heuristic* unexpectedly yielded one of the most equitable outcomes. Upon further inspection, this result stemmed from a coincidental advantage: in two instances, the method selected an asset for removal from a location where **two ambulances were co-located**, leaving the second asset in place and thereby retaining service in that area. While unintentional, this highlights how simple or naive heuristics can sometimes produce favorable configurations by chance, especially in systems with overlapping asset placement.

This scenario illustrates that even under minimal stress conditions, different model assumptions lead to markedly different outcomes in both coverage and equity. Moreover, it underscores the possibility that simple heuristics may, by chance, outperform more sophisticated models in specific contexts, although such outcomes are unlikely to generalize.

6.4.2 Scenario 2: Mixed Population Density Extended MIP Activation (20 Assets Reallocated - Extended-MIP)

In the second scenario, the number of reallocated assets was increased to 20, simulating a moderate-level surge activation. This larger intervention introduced more visible strain on system performance, and more distinct differences emerged between models.

Coverage Outcomes

A greater number of models failed to maintain full coverage. Specifically, the *Skip-One heuristic*, the *2-on, 1-off heuristic*, and the *Maximal Covering Location Problem (MCLP)* were all unable to preserve full demand coverage. By contrast, the remaining models, including *MEXCLP* and *BACOP II*, successfully retained full geographic coverage, demonstrating more resilience under increased stress.

The underperformance of MCLP is particularly noteworthy, as it had succeeded in Scenario 1. Its prioritization of *maximizing total population covered* may lead to the prioritization of urban (highly populated) areas and neglect of more rural demand, especially under tighter resource constraints.

Equity Outcomes

Equity results further highlighted differences in model behavior. The *2-on, 1-off heuristic*, which previously had one of the best equity performances, now produced a **more unequal distribution** of coverage, reflected in a higher Gini coefficient. This reinforces the hypothesis that its earlier success was largely circumstantial.

Interestingly, the *Skip-One heuristic*, which had underperformed in Scenario 1, now achieved a relatively better distribution. This was due to its accidental selection of assets from locations that remained partially covered by other units, thereby preserving some redundancy.

Contrasting sharply with its earlier performance, the *BACOP I model* yielded the **least equitable coverage** in Scenario 2, recording the highest Gini coefficient of all tested models. This indicates a high sensitivity to the scale of asset reallocation: while BACOP I may offer equity under minimal constraints, its assumptions break down under more aggressive surge activation.

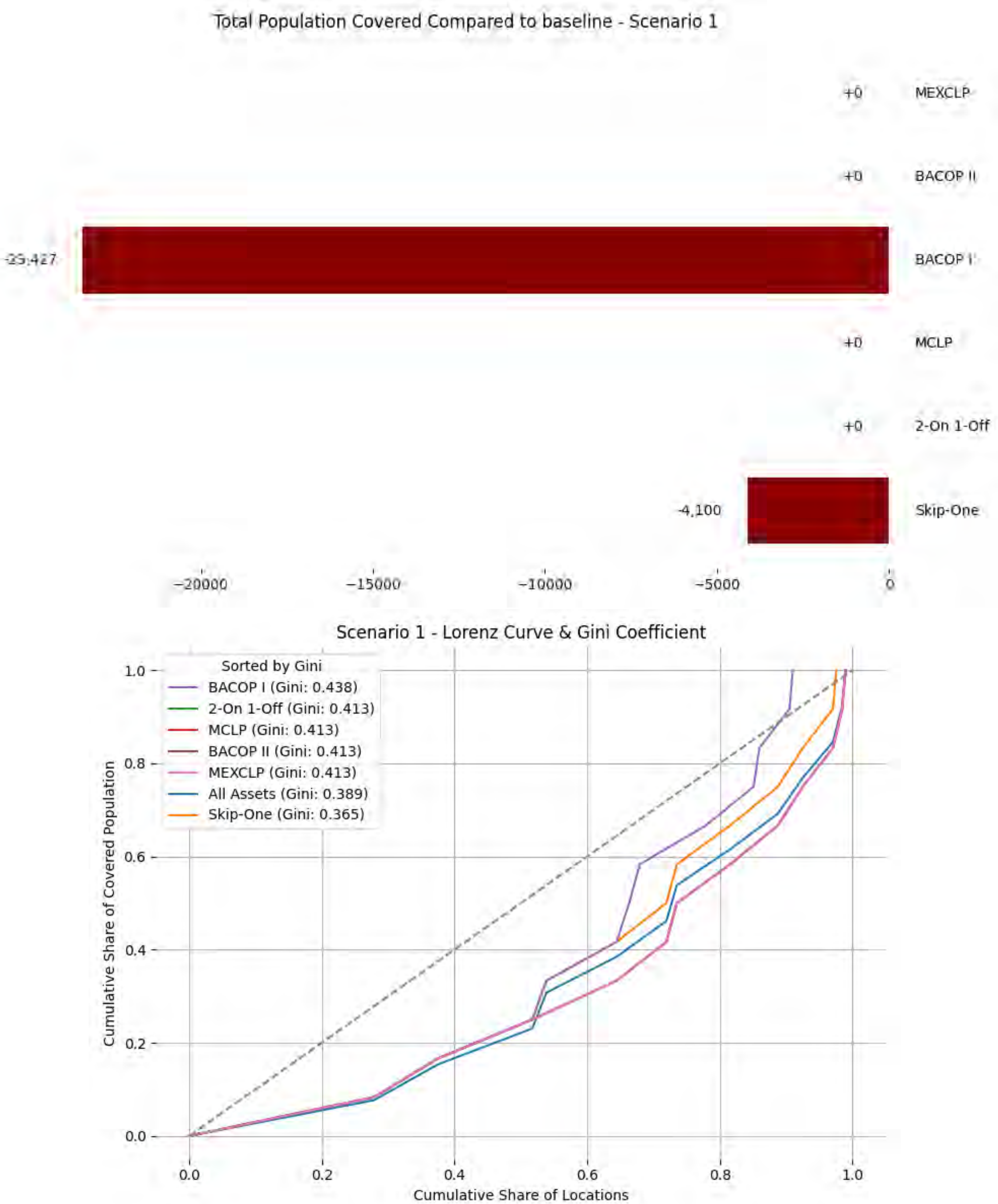


Figure 6.6: Population coverage (top) and Lorenz Curve & Gini Coefficient (bottom) - Scenario 1

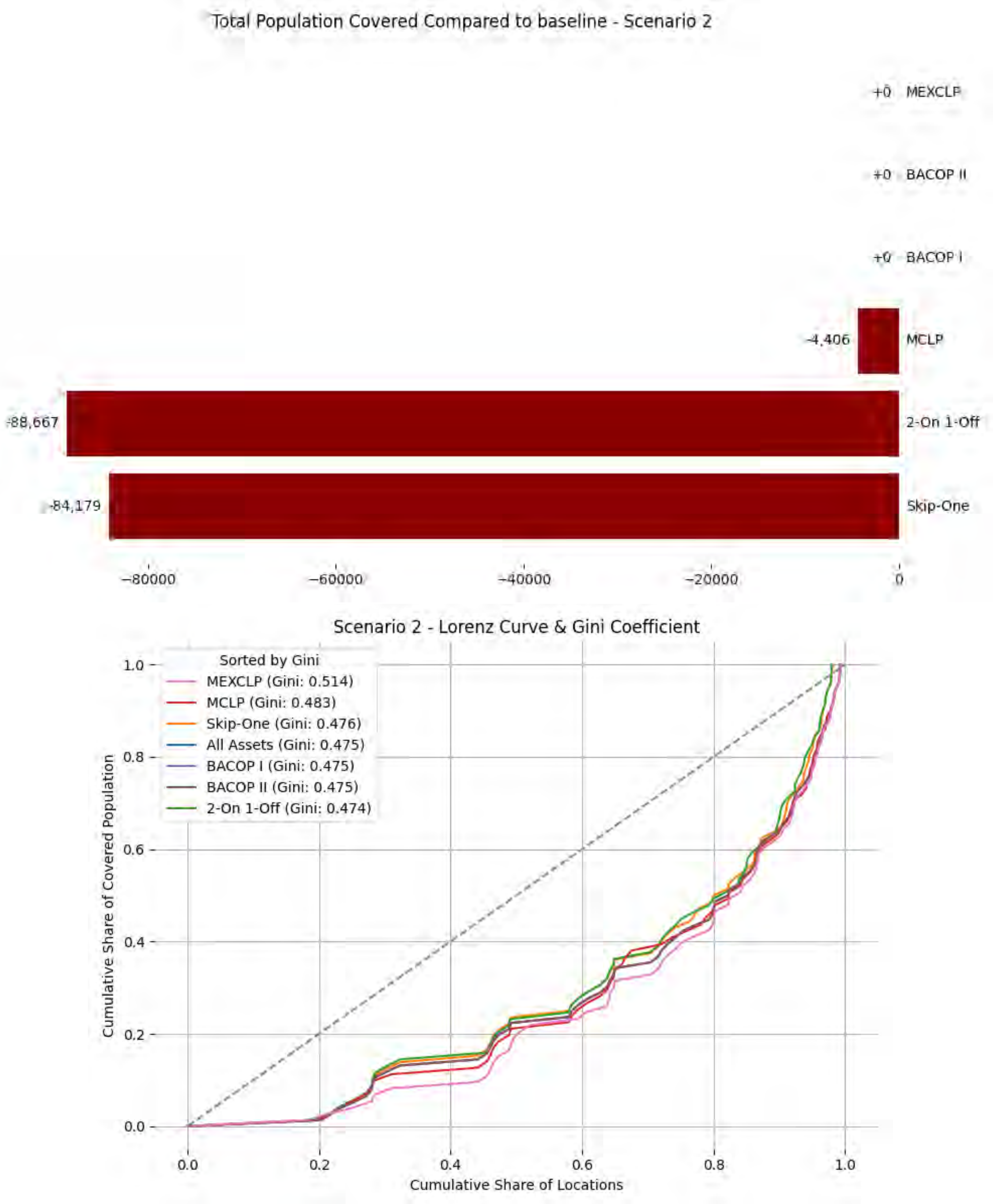


Figure 6.7: Population coverage (top) and Lorenz Curve & Gini Coefficient (bottom) - Scenario 2

6.4.3 Scenario 3: Urban Extended MIP Activation (20 Assets Reallocated - Extended-MIP)

Scenario 3 simulates a surge event in a highly urbanized area under the *extended MIP protocol*, allowing models to draw resources not only from the affected province but also from neighboring provinces. This broader pool of assets enables a richer redistribution, especially for optimization-based models.

Coverage Outcomes

All models successfully maintained full coverage of demand within the affected urban region, except for the *2-on, 1-off heuristic*. This was expected, as assets in urban centers tend to serve smaller municipalities with dense populations, increasing their local coverage efficiency. Removing such assets disproportionately affects the ability to meet local demand.

Equity Outcomes

Equity analysis revealed clear differences between heuristics and formal optimization models. The commonly used *Skip-One* and *2-on, 1-off* heuristics both resulted in **higher Gini coefficients** than the baseline, indicating more unequal coverage. This was expected given their inability to account for population density when selecting assets for reallocation.

By contrast, all other models, except for *MCLP*, produced more equitable distributions than the baseline. The improvement arises from their ability to reallocate assets in a way that smooths coverage across population centers. MCLP, while maintaining full coverage, selected assets primarily from *rural regions*, leading to a slight imbalance favoring urban areas. This is consistent with the model's objective of maximizing total population coverage, often at the cost of equity.

6.4.4 Scenario 4: Rural Maxi-MIP Activation (40 Assets Reallocated - Maxi-MIP)

In this scenario, a significant reallocation of **40 ambulance assets** was simulated, creating widespread strain on the system, particularly in rural zones.

Coverage Outcomes

As with prior scenarios, the two heuristics (*Skip-One* and *2-on, 1-off*) failed to maintain full coverage, especially in regions close to the simulated mass casualty incident (MCI) zone, from which the assets were drawn. Their naive selection approach, focused on local removal, resulted in large uncovered zones.

The optimization models distributed asset reallocation across the national landscape. Notably, *MCLP* deliberately sacrificed coverage in sparsely populated rural areas to preserve urban coverage. While this trade-off allowed it to maintain high total population coverage, it came at a significant cost to equity.

Equity Outcomes

The heuristics again produced the worst Gini coefficients, reflecting a highly unequal spread of EMS coverage. Interestingly, while MCLP retained broad coverage, its equity performance declined sharply, resulting in one of the lowest Lorenz curves of all models. This reinforces its tendency to prioritize dense population areas at the expense of rural zones.

Other models, particularly *BACOP II* and *MEXCLP*, produced more balanced solutions, retaining both broad geographic coverage and improved distributional fairness.

6.4.5 Scenario 5: Urban Maxi-MIP Activation (40 Assets Reallocated - Maxi-MIP)

The final scenario simulates a large-scale asset reallocation in response to a remote MCI, compounding the stress on already stretched EMS resources.

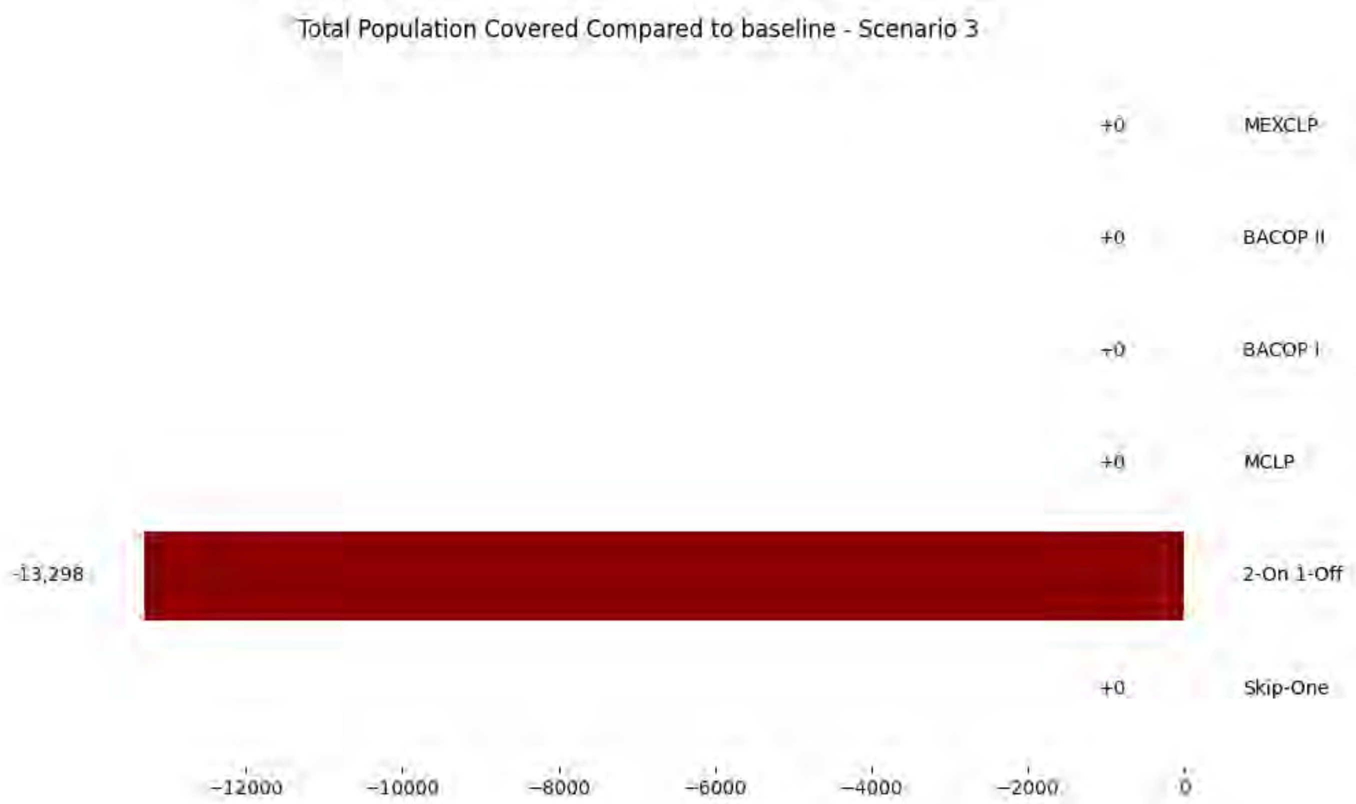
Coverage Outcomes

As observed in Scenarios 2 and 4, the *Skip-One*, *2-on, 1-off*, and *MCLP* models failed to preserve full demand coverage. In this case, the large number of assets removed, often from critical zones, created substantial coverage gaps.

Equity Outcomes

Equity analysis revealed that, among all models, only *MCLP* performed **worse than the baseline configuration** with all assets available. While its population-maximization approach still benefited dense urban clusters, the exclusion of low-density areas resulted in a highly skewed distribution.

All remaining models yielded Gini coefficients lower than baseline, indicating improved fairness in coverage despite the reduced number of available assets. This again underscores the importance of incorporating spatial and demographic fairness into asset selection criteria during surge activation.



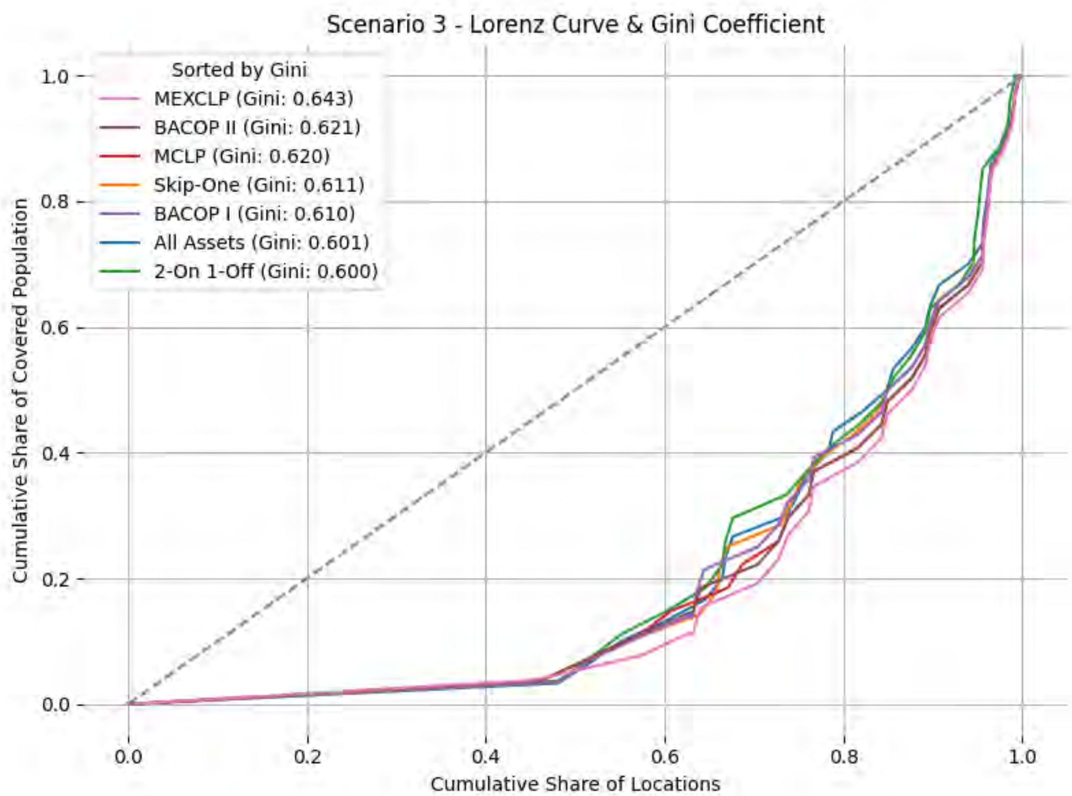
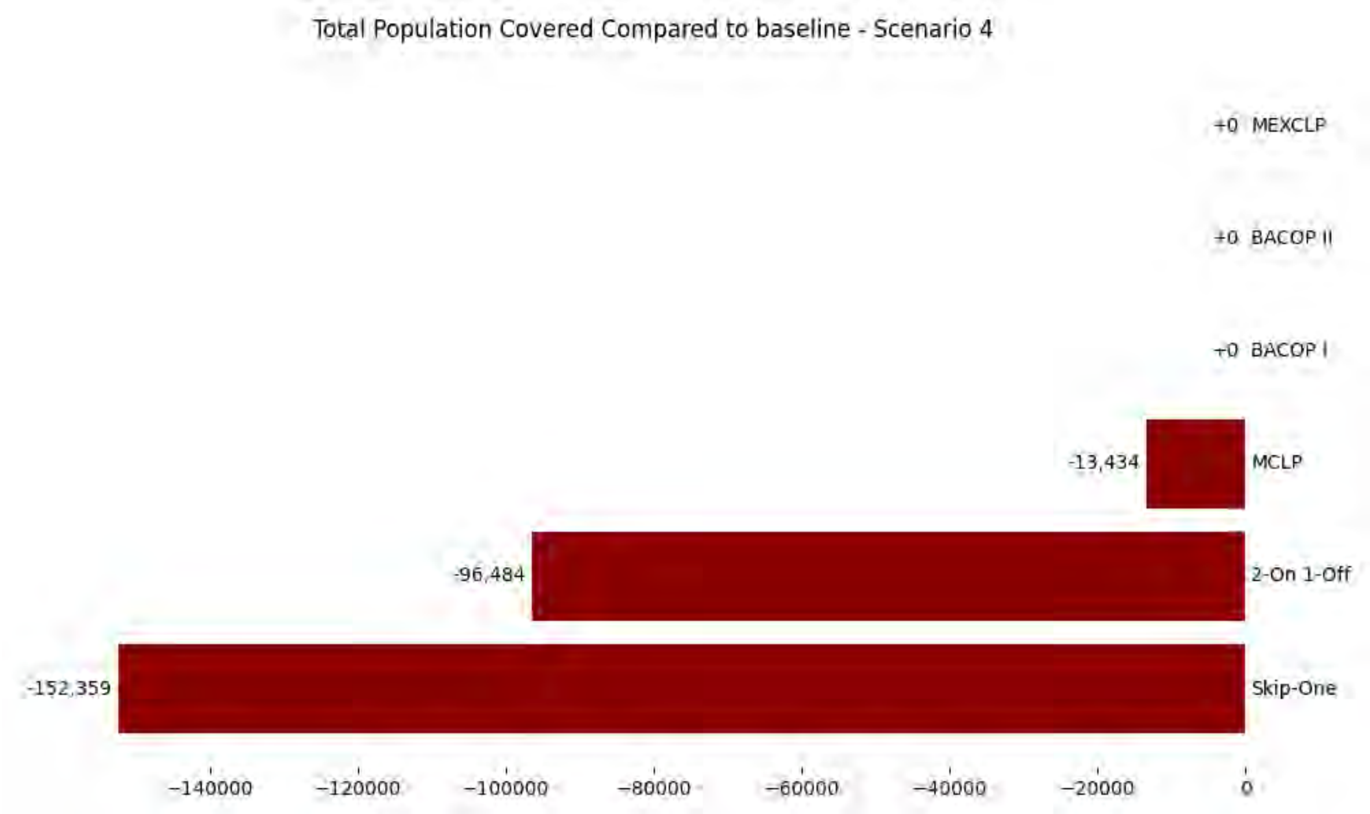


Figure 6.8: Population coverage (top) and Lorenz Curve & Gini Coefficient (bottom) - Scenario 3



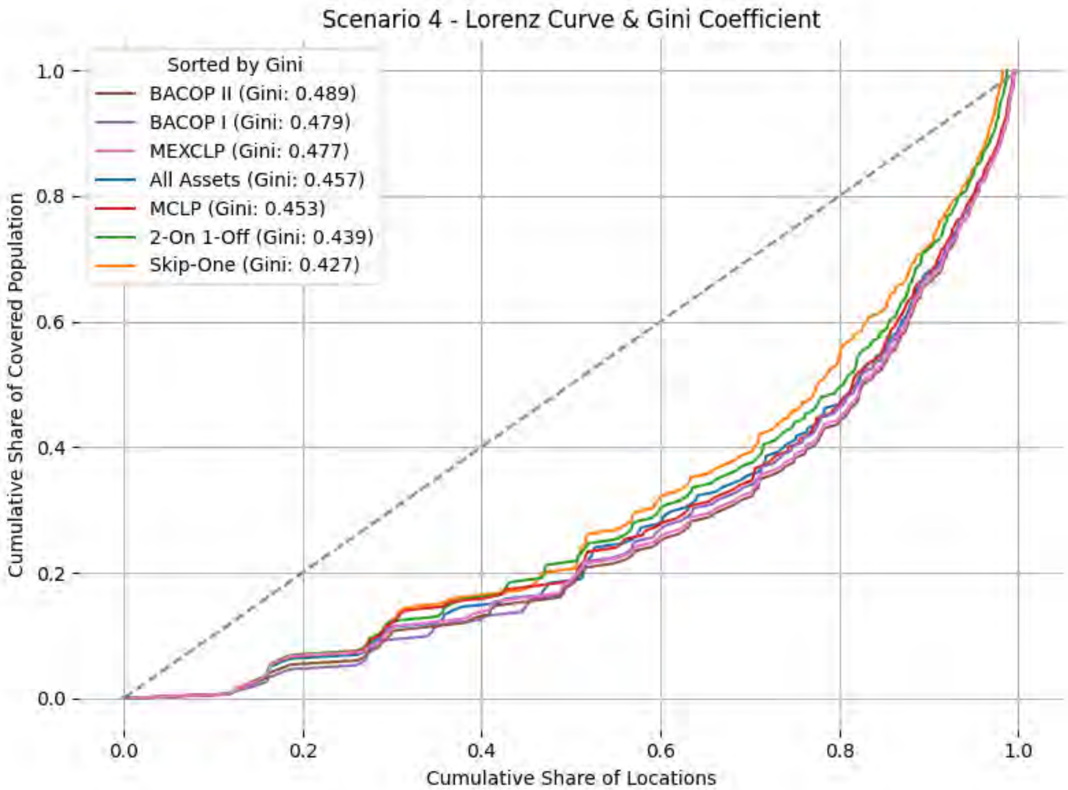


Figure 6.9: Population coverage (top) and Lorenz Curve & Gini Coefficient (bottom) - Scenario 4

7 Discussion

This chapter critically evaluates the findings of this thesis in relation to the stated research questions:

1. the comparative strengths and limitations of deterministic, probabilistic, and dynamic location models in surge EMS planning;
2. the ability of location models to identify optimal subsets of EMS assets for reallocation amid constrained baseline coverage;
3. practical considerations for implementation in the Belgian context;
4. the role of region-specific variables in influencing EMS demand and model applicability.

Each of these aspects is discussed in the context of both the empirical simulation results and the broader literature.

7.1 Interpretation of Key Results

The scenario-based simulation conducted in this thesis consistently demonstrated that formal, optimization-based location models outperformed current heuristic practices employed in Belgian EMS asset reallocation. Models such as the MCLP, BACOP I, BACOP II, and the MEXCLP delivered superior outcomes in terms of both population coverage and equity, especially as the scale and complexity of reallocation increased. Specifically, even the earliest deterministic models regularly surpassed the “Skip-One” and “2-on, 1-off” heuristics, particularly under progressively severe surge scenarios (Extended and MAXI-MIP).

A notable and robust pattern was the differential performance of models with respect to geographic and demographic equity. The Gini coefficient, adopted in this thesis as a quantitative measure of equity, was substantially lower for optimization models in most crisis scenarios, indicating a more just distribution of EMS coverage as shown in Gini coefficients for each scenario and model. Heuristic approaches, by contrast, often failed to preserve even minimum levels of coverage, especially in rural or mixed-density scenarios. These findings suggest that structured, theoretically grounded models can simultaneously sustain critical levels of population coverage and promote resource fairness, two objectives that are frequently presented as mutually exclusive in the operational literature.

The study also confirmed that deterministic models, despite their inherent limitations, remain powerful tools in resource-constrained settings such as Belgium’s. While models like MCLP tend to prioritize absolute coverage (often favoring urban centers), they nonetheless maintain higher aggregate equity than ad hoc methods. The inclusion of probabilistic elements, as in MEXCLP and BACOP II, addressed certain shortcomings of deterministic approaches, namely, by explicitly accounting for backup needs and the probabilistic unavailability of assets. These models proved particularly effective in large-scale and urban surge scenarios (e.g., scenarios 3 and 5), underscoring the value of reliability-focused planning.

Interestingly, a few unexpected findings emerged from the simulation. For instance, in the lowest-intensity scenario (Scenario 1), some heuristics delivered greater equity, as measured by the Gini coefficient, despite not ensuring full geographic coverage. Upon inspection, this was attributable to artifact, namely, the serendipitous retention of coverage in select areas with co-located assets. This phenomenon supports the central claim of the thesis: that primitive heuristics may appear to “work” in trivial cases due to network redundancy, but lack the generalizability, transparency, and replicability required for evidence-based policy.

7.2 Comparison to Prior Literature

The superiority of theoretically grounded optimization models over simple heuristics is well supported in the global literature. Studies across North America and the UK, for example, have long demonstrated the utility of MCLP and MEXCLP variants in providing both immediate and backup coverage in urban and rural contexts [14, 23, 24]. The present thesis extends these findings to a previously understudied European context characterized by a decentralized, multi-tiered EMS system and resource scarcities.

Consistent with observations in the US and Canada, the BACOP family of models performed especially well in maintaining service equity during high-stress, resource-limited situations. The advantages of multi-coverage and explicit backup planning identified here parallel those articulated by Hogan and ReVelle in 1986 and subsequent systematic reviews

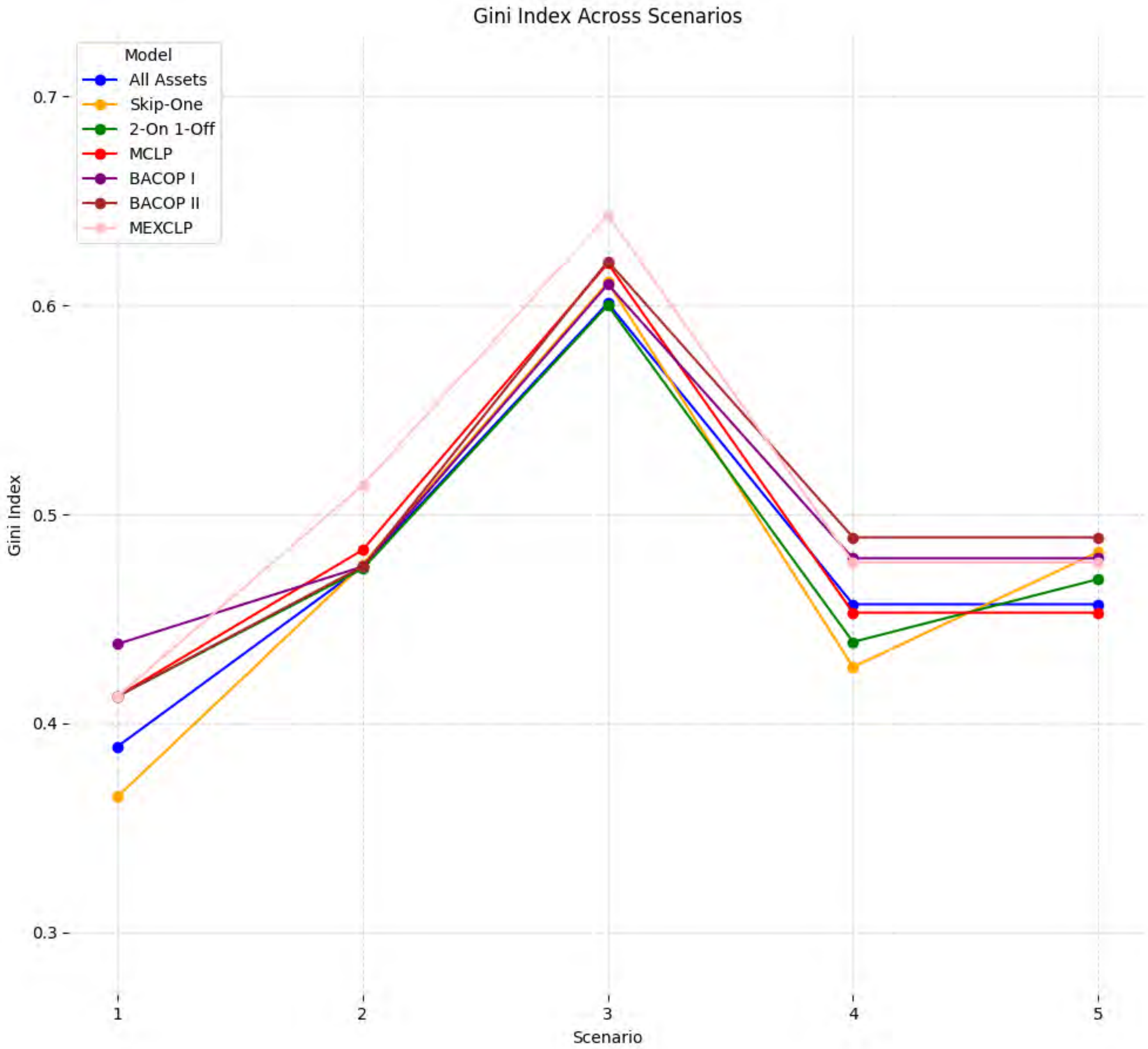


Figure 7.1: Gini coefficients for each scenario and model.

[16, 22]. The empirical evidence herein affirms such findings: BACOP I and II remained robust against equity erosion even as asset removal intensified, unlike heuristics that sacrificed rural and peri-urban regions for convenience.

Yet, the implementation gap between theory and Belgian practice, highlighted throughout the thesis, mirrors a persistent theme in the EMS modeling literature. The absence of a unified, evidence-based resource allocation framework in Belgium echoes experiences in other countries prior to model adoption [8, 35]. Moreover, the observed trade-offs when using models that prioritize maximum population coverage (e.g., MCLP) versus those favoring backup and reliability (e.g., BACOPs and MEXCLP) reinforce established theoretical debates regarding "coverage maximization" versus "reliability assurance" as rival paradigms [27].

The thesis also substantiates critiques of deterministic models. As others have warned, static models risk privileging efficiency over resilience, sometimes failing to account for simultaneous calls, asset unavailability, or demand surges, problems that are especially salient in the Belgian system given its prohibitions on excess baseline capacity [17]. The marginal benefits observed for probabilistic approaches in the simulation echo international calls for stochastic or dynamic extensions [28, 30].

7.3 Explanation of Limitations and Unexpected Results

Several limitations inherent to the study's design must be acknowledged. First, the simulation design, while more sophisticated than most Belgian practice, still relies on municipality-level centroids as demand proxies. Such spatial aggregation risks obscuring intra-municipal disparities and border effects, potentially over- or underestimating practical coverage. Second, all models utilized a single-tier (ambulance-only) abstraction, omitting the PIT and MUG components that play vital roles in the real Belgian EMS system. This was a necessary simplification, dictated by data and software constraints, but nonetheless narrows the scope of directly actionable recommendations.

A further significant limitation is the reliance on population counts as a demand proxy. The literature reviewed in this thesis demonstrates that EMS call volume is shaped by age structure, socioeconomic status, event-driven factors, and temporal variability [34, 40]. The unavailability of granular, real-time, or patient-level Belgian EMS data constrained this study's ability to refine demand estimation. Consequently, while population-based modeling is a pragmatic first step, accepted in foundational studies, future implementation should prioritize more nuanced, empirically supported demand predictors.

Another methodological challenge is the static modeling of both demand and travel times. The lack of Belgian fleet telematics, real-time hospital queue data, and dynamic routing further precluded simulation of dynamic or real-time optimization models. As discussed in the thesis, these advanced approaches (e.g., time-staged integer programming, dynamic floating stations) are increasingly prevalent in other health systems, yet are currently infeasible under Belgium's technical, regulatory, and infrastructural landscape. The implications of excluding these models are significant: full realization of efficiency and equity potential likely requires meaningful investment in data and IT infrastructure.

Finally, the simulation's use of Gini coefficients, while innovative for the EMS logistics literature, is not without challenges. While the Gini coefficient is a well-accepted measure of inequality in economics, its application to spatial healthcare coverage is relatively novel. Further research should validate this metric against clinical or operational outcomes.

7.4 Policy and Practical Implications

Despite these acknowledged limitations, the findings of this thesis have clear and actionable implications for EMS planners and policymakers in Belgium and similar contexts. First and foremost, the consistent outperformance of established location optimization models over ad hoc heuristics across all but the most trivial surge scenarios makes a compelling case for at least partial adoption in preparedness planning and real-world crisis response.

Second, the study demonstrates that even deterministic models, if transparently applied and systematically updated, can substantially improve both population coverage and equity, particularly in urban and peri-urban settings. Immediate integration of such models within GIS-based planning tools and national/regional drills could yield meaningful improvements in both baseline resilience and surge capacity.

Third, the simulation highlights the risks of over-relying on maximal coverage models (such as MCLP) in highly diverse geographic settings. Policymakers must recognize that "maximize covered population" and "maximize coverage fairness" are not always aligned objectives, especially where rural communities risk systematic underprotection. The BACOP and MEXCLP models, by explicitly weighting redundancy, offer promising pathways to balancing efficiency and equity, an increasingly urgent goal given the social and ethical dimensions of EMS delivery.

Finally, the results suggest that prior to any wide-scale roll-out, targeted pilots and retrospective analysis of past surge incidents should be pursued. The translation of model recommendations into operational protocols will require stakeholder buy-in, technical training, and iterative evaluation to address both technical and cultural barriers.

7.5 Directions for Future Research

The present research opens several avenues for future inquiry, both within Belgium and internationally:

1. **Integration of Granular Demand Predictors**: Further empirical research is needed to systematically identify the true determinants of EMS demand in Belgium, including age, socioeconomic status, seasonal patterns, and event-driven spikes. Direct linkage with real operational and patient-level data remains a priority.
2. **Multi-tier Model Implementation**: Extension of the present simulation framework, currently single-tier (ambulance only), to multi-tiered asset models (TEAM, FLEET, PROFLEET) is crucial. This would better reflect the actual complexity and interdependencies of Belgian EMS and allow evaluation of tier-specific as well as system-wide optimization strategies.
3. **Dynamic and Real-time Models**: Future studies should revisit dynamic and time-sensitive models if and when Belgian EMS infrastructure evolves to provide real-time asset tracking and interoperable datasets. The international literature suggests substantial gains may be achievable in both response times and operational efficiency.
4. **Prospective Pilot Studies and Stakeholder Engagement**: Before full implementation, prospective pilots involving model-driven asset reallocation in controlled settings, ideally with randomized or quasi-experimental designs, should be undertaken. These pilots must involve rigorous engagement with frontline EMS workers, dispatchers, and regional authorities to ensure practical feasibility.
5. **Validation of Equity Metrics**: Further exploration of how metrics like the Gini coefficient correlate with relevant operational outcomes (e.g., morbidity, mortality, patient satisfaction) will help to legitimize and refine the use of such indicators.

6. **Policy-Systems-Fit Analysis**: Research into the regulatory, funding, and organizational adaptations necessary to enable model-driven EMS planning in Belgium's mixed public-private environment is warranted. Cross-national comparisons, especially with other semi-public European systems, could illuminate best practices and transferable lessons.

7.6 Conclusion

This thesis demonstrates that structured, transparent, and well-studied EMS location models can deliver immediate and substantial improvements over current heuristic methods in Belgium, especially with respect to equity and resilience under surge conditions. While limitations in data and multi-tier realism persist, and the full adoption of dynamic approaches remains aspirational, there now exists a robust theoretical and empirical basis for beginning the transition toward evidence-based EMS asset allocation. Future research and policy should focus on building the necessary data infrastructure, incentivizing pilot adoption, and continuously evaluating both efficiency and fairness of EMS delivery in increasingly complex and resource-constrained environments.

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Appendices - Code examples

Appendix 1 - Example code BACOP I

The following python code was used to solve the BACOP I example:

```
from pulp import *

# Common parameters for BACOP I
I = range(1, 7)  # 6 demand points
J = ['A', 'B', 'C']  # 3 candidate locations
p = 2  # number of facilities to place

# Parameters
a = {1: 100, 2: 200, 3: 150, 4: 120, 5: 180, 6: 250}  # population at each demand point
S = {1: ['A'], 2: ['A'], 3: ['A', 'B'], 4: ['B'], 5: ['B', 'C'], 6: ['C']}  # coverage sets

# BACOP I Model
def solve_bacop_i():
    prob = LpProblem("BACOP_I_example", LpMaximize)

    # Define decision variables
    x = LpVariable.dicts("x", J, 0, 1, LpBinary)
    y = LpVariable.dicts("y", I, 0, 1, LpBinary)
    z = LpVariable.dicts("z", I, 0, 1, LpBinary)

    # Define the objective function
    prob += lpSum(a[i] * z[i] for i in I)

    # Define the constraints
    for i in I:
        prob += lpSum(x[j] for j in S[i]) >= y[i] + z[i]
        prob += y[i] >= z[i]

    # Facility location constraint
    prob += lpSum(x[j] for j in J) == p

    # Solve the problem
    prob.solve()

    # Print the results
    print("BACOP I Results:")
    print("Status:", LpStatus[prob.status])
    print("Objective value:", value(prob.objective))
    print("\nFacility Locations:")
    placed_facilities = [j for j in J if value(x[j]) > 0.5]
    for j in placed_facilities:
        print(f"Facility located at {j}")

    print("\nCoverage:")
    for i in I:
        # Calculate coverage directly from placed facilities
        covering_facilities = [j for j in S[i] if j in placed_facilities]
        primary = len(covering_facilities) >= 1
        backup = len(covering_facilities) >= 2

        print(f"Demand point {i}:")
        print(f"  Primary coverage: {primary}")
        print(f"  Backup coverage: {backup}")
        print(f"  Facilities covering: {covering_facilities}\n")


# Solve BACOP I
solve_bacop_i()
```

Appendix 2 - Example code BACOP II

The following python code was used to solve the BACOP II example:

```python
from pulp import *

# Common parameters for BACOP II
I = range(1, 7)  # 6 demand points
J = ['A', 'B', 'C']  # 3 candidate locations
p = 2  # number of facilities to place

# Parameters
a = {1: 100, 2: 200, 3: 150, 4: 120, 5: 180, 6: 250}  # population at each demand point
S = {1: ['A'], 2: ['A'], 3: ['A', 'B'], 4: ['B'], 5: ['B', 'C'], 6: ['C']}  # coverage sets

# BACOP II Model
def solve_bacop_ii():
    prob = LpProblem("BACOP_II_example", LpMaximize)

    # Define decision variables
    x = LpVariable.dicts("x", J, 0, 1, LpBinary)
    y = LpVariable.dicts("y", I, 0, 1, LpBinary)
    r = LpVariable.dicts("r", I, 0, 1, LpBinary)

    # Define the objective functions (weighted sum approach)
    w1, w2 = 0.5, 0.5  # Adjust weights as needed
    prob += w1 * lpSum(a[i] * y[i] for i in I) + w2 * lpSum(a[i] * r[i] for i in I)

    # Define the constraints
    for i in I:
        prob += lpSum(x[j] for j in S[i]) >= y[i] + r[i]
        prob += y[i] >= r[i]

    # Facility location constraint
    prob += lpSum(x[j] for j in J) == p

    # Solve the problem
    prob.solve()

    # Print the results
    print("BACOP II Results:")
    print("Status:", LpStatus[prob.status])
    print("Objective value:", value(prob.objective))
    print("\nFacility Locations:")
    placed_facilities = [j for j in J if value(x[j]) > 0.5]
    for j in placed_facilities:
        print(f"Facility located at {j}")

    print("\nCoverage:")
    for i in I:
        # Calculate coverage directly from placed facilities
        covering_facilities = [j for j in S[i] if j in placed_facilities]
        primary = len(covering_facilities) >= 1
        backup = len(covering_facilities) >= 2

        print(f"Demand point {i}:")
        print(f"  Primary coverage: {primary}")
        print(f"  Backup coverage: {backup}")
        print(f"  Facilities covering: {covering_facilities}\n")


# Solve BACOP II
solve_bacop_ii()
```

Appendix 3 - Example code DSM

The following python code was used to solve the DSM example.

```python
from pulp import *

# DSM Example Parameters (8 demand points, 4 locations, 3 ambulances)
I = range(1, 9)  # Demand points 1-8
J = ['A', 'B', 'C', 'D']  # Potential locations
p = 3  # Total ambulances to place
alpha = 0.8  # Minimum proportion for r1 coverage
r1_coverage = 2  # Units for double coverage
r2_coverage = 4  # Units for universal coverage

# Coverage matrices (a_ij for r1, delta_ij for r2)
a_ij = {
    1: {'A': 1, 'B': 0, 'C': 0, 'D': 0},
    2: {'A': 1, 'B': 1, 'C': 0, 'D': 0},
    3: {'A': 0, 'B': 1, 'C': 0, 'D': 0},
    4: {'A': 0, 'B': 1, 'C': 1, 'D': 0},
    5: {'A': 0, 'B': 0, 'C': 1, 'D': 0},
    6: {'A': 0, 'B': 0, 'C': 1, 'D': 1},
    7: {'A': 0, 'B': 0, 'C': 0, 'D': 1},
    8: {'A': 0, 'B': 0, 'C': 0, 'D': 1}
}

delta_ij = {
    1: {'A': 1, 'B': 1, 'C': 0, 'D': 0},
    2: {'A': 1, 'B': 1, 'C': 1, 'D': 0},
    3: {'A': 1, 'B': 1, 'C': 1, 'D': 0},
    4: {'A': 0, 'B': 1, 'C': 1, 'D': 1},
    5: {'A': 0, 'B': 1, 'C': 1, 'D': 1},
    6: {'A': 0, 'B': 0, 'C': 1, 'D': 1},
    7: {'A': 0, 'B': 0, 'C': 1, 'D': 1},
    8: {'A': 0, 'B': 0, 'C': 0, 'D': 1}
}


def solve_dsm():
    prob = LpProblem("Double_Standard_Model", LpMaximize)

    # Decision variables
    y = LpVariable.dicts("Ambulances", J, lowBound=0, upBound=2, cat='Integer')
    x1 = LpVariable.dicts("SingleCoverage", I, cat='Binary')
    x2 = LpVariable.dicts("DoubleCoverage", I, cat='Binary')

    # Objective: Maximize double coverage within r1
    prob += lpSum(x2[i] for i in I)

    # Universal coverage constraint (r2)
    for i in I:
        prob += lpSum(delta_ij[i][j] * y[j] for j in J) >= 1

    # Proportional coverage constraint (r1)
    prob += lpSum(x1[i] + x2[i] for i in I) >= alpha * len(I)

    # Coverage linkage constraints
    for i in I:
        prob += lpSum(a_ij[i][j] * y[j] for j in J) >= x1[i] + 2 * x2[i]
        prob += x2[i] <= x1[i]

    # Resource constraints
    prob += lpSum(y[j] for j in J) == p
    for j in J:
        prob += y[j] <= 2
```

```
    # Solve the problem
    prob.solve()

    # Print results
    print("\nDSM Results:")
    print("Status:", LpStatus[prob.status])
    print("Objective value (Double coverage):", value(prob.objective))

    print("\nAmbulance Placement:")
    for j in J:
        print(f"Location {j}: {int(value(y[j]))} ambulances")

    print("\nCoverage Analysis:")
    for i in I:
        r1_ambulances = sum(a_ij[i][j] * value(y[j]) for j in J)
        print(f"Demand Point {i}:")
        print(f"  r1 Coverage: {r1_ambulances} ambulances")
        print(f"  Single Coverage: {value(x1[i])}")
        print(f"  Double Coverage: {value(x2[i])}")


# Solve the DSM problem
solve_dsm()
```

Appendix 4 - Example code MEXCLP

The following python code was used to solve the MEXCLP example.

```
from pulp import *

# Common parameters for MEXCLP
I = range(1, 6)  # Demand points
J = ['A', 'B', 'C']  # Candidate locations
p = 2  # Maximum coverage level
M = 2  # Number of facilities to place

# Parameters
q = 0.3  # Busy probability
d = {1: 100, 2: 80, 3: 120, 4: 60, 5: 90}  # Demand
S = {1: ['A', 'B'], 2: ['A', 'C'], 3: ['B', 'C'], 4: ['B'], 5: ['C']}  # Coverage sets

# MEXCLP Model
def solve_MEXCLP():
    prob = LpProblem("MEXCLP_example", LpMaximize)

    # Define decision variables
    x = LpVariable.dicts("x", J, 0, 1, LpBinary)
    y = LpVariable.dicts("y", [(i, k) for i in I for k in range(1, p + 1)], 0, 1, LpBinary)

    # Define the objective function
    prob += lpSum(d[i] * (1-q) * q**(k-1) * y[(i, k)] for i in I for k in range(1, p+1))

    # Define the constraints
    for i in I:
        prob += lpSum(x[j] for j in S[i]) >= lpSum(y[(i, k)] for k in range(1, p + 1))

    # Facility location constraint
    prob += lpSum(x[j] for j in J) == M

    # Logical constraints
    for i in I:
        for k in range(1, p):
            prob += y[(i, k + 1)] <= y[(i, k)]

    # Solve the problem
    prob.solve()

    # Print the results
    print("MEXCLP Results:")
    print("Status:", LpStatus[prob.status])
    print("Objective value:", value(prob.objective))
    print("\nFacility Locations:")
    placed_facilities = [j for j in J if value(x[j]) > 0.5]
    for j in placed_facilities:
        print(f"Facility located at {j}")

    print("\nCoverage:")
    for i in I:
        coverage_level = sum(1 for k in range(1, p + 1) if value(y[(i, k)]) > 0.5)
        print(f"Demand point {i} is covered by {coverage_level} facilities")

# Solve MEXCLP
solve_MEXCLP()
```

Appendix 5 - Example code PLSCP

```python
from pulp import *

def solve_plscp():
    # Problem parameters
    demand_points = range(1, 7)  # 6 demand points
    stations = ['A', 'B', 'C']  # 3 candidate stations
    coverage_sets = {
        1: ['A', 'B'],
        2: ['A', 'C'],
        3: ['B', 'C'],
        4: ['B', 'C'],
        5: ['B'],
        6: ['B']
    }
    F_i = {
        1: 7, 2: 7, 3: 7,
        4: 7, 5: 6, 6: 1
    }
    alpha = 0.9

    # Calculate minimum required ambulances (b_i)
    def calculate_b_i(F, alpha):
        b = 1
        while True:
            if 1 - (F / b) ** b >= alpha:
                return b
            b += 1
            if b > 100:  # Fail-safe for convergence
                raise ValueError("No solution found for F=%.1f, alpha=%.2f" % (F, alpha))

    b = {i: calculate_b_i(F_i[i], alpha) for i in demand_points}

    # Create optimization model
    model = LpProblem("PLSCP_EMS_Example", LpMinimize)

    # Decision variables (ambulances per station)
    x = LpVariable.dicts("x", stations, lowBound=0, cat='Integer')

    # Objective function: Minimize total ambulances
    model += lpSum(x[s] for s in stations)

    # Coverage constraints
    for i in demand_points:
        covering_stations = coverage_sets[i]
        model += lpSum(x[s] for s in covering_stations) >= b[i]

    # Solve model
    model.solve(PULP_CBC_CMD(msg=False))

    # Output results
    print("PLSCP Solution Results:")
    print(f"Status: {LpStatus[model.status]}")
    print(f"Total ambulances required: {int(value(model.objective))}\n")

    print("Station deployments:")
    for s in stations:
        print(f"- Station {s}: {int(value(x[s]))} ambulances")

    print("\nCoverage requirements:")
    for i in demand_points:
        print(f"Demand point {i}: Requires {b[i]} ambulances in {coverage_sets[i]}")
```

```python
# Execute the solver
solve_plscp()
```

Appendix 6 - Example code MALP I

The following python code was used to solve the MALP I example.

```
from pulp import *

# MALP I Parameters
I = range(1, 7)  # 6 demand nodes
J = range(1, 7)  # 6 potential ambulance locations
B = 3  # number of ambulances to place
a = {1: 100, 2: 150, 3: 120, 4: 130, 5: 110, 6: 140}  # demand at each node
q = 0.3  # busy probability
alpha = 0.7  # minimum availability probability

# Coverage sets (locations covering each demand node within response time)
N = {
    1: [1, 2],
    2: [1, 2, 3],
    3: [2, 3, 4],
    4: [3, 4, 5],
    5: [4, 5, 6],
    6: [5, 6]
}

def solve_malp_i():
    # Create optimization problem
    prob = LpProblem("MALP_I_Implementation", LpMaximize)

    # Decision variables
    x = LpVariable.dicts("x", J, 0, 1, LpBinary)  # ambulance placement
    y = LpVariable.dicts("y", I, 0, 1, LpBinary)  # coverage status

    # Objective function: Maximize covered demand
    prob += lpSum(a[j] * y[j] for j in I)

    # Calculate required ambulances for probability constraint
    m = 1
    while 1 - q**m < alpha:
        m += 1

    # Constraints
    for j in I:
        # Coverage requires at least m ambulances in coverage set
        prob += lpSum(x[i] for i in N[j]) >= m * y[j]

    # Ambulance fleet size constraint
    prob += lpSum(x[i] for i in J) == B

    # Solve
    prob.solve()

    # Print results
    print("MALP I Results:")
    print(f"Status: {LpStatus[prob.status]}")
    print(f"Objective Value: {value(prob.objective)}\n")

    print("Ambulance Locations:")
    placed = [i for i in J if value(x[i]) > 0.5]
    for loc in placed:
        print(f"  - Location {loc}")

    print("\nCoverage Status:")
    for j in I:
        print(f"Node {j}: {'Covered' if value(y[j]) > 0.5 else 'Not Covered'}")
```

```
# Execute the solver
solve_malp_i()
```

Appendix 7 - Example code MALP II using PuLP

The following python code was used to solve the MALP II example, using the approximation method and the PuLP-library.

```
from pulp import *
import math

# MALP II Parameters
I = range(1, 7)  # 6 demand nodes
J = range(1, 7)  # 6 potential ambulance locations
B = 3  # number of ambulances to place
a = {1: 100, 2: 150, 3: 120, 4: 130, 5: 110, 6: 140}  # demand at each node
q_i = {1: 0.25, 2: 0.3, 3: 0.35, 4: 0.2, 5: 0.4, 6: 0.15}  # location-specific busy probabilities
alpha = 0.7  # minimum availability probability

# Coverage sets (locations covering each demand node within response time)
N = {
    1: [1, 2],
    2: [1, 2, 3],
    3: [2, 3, 4],
    4: [3, 4, 5],
    5: [4, 5, 6],
    6: [5, 6]
}


def solve_malp_ii():
    # Create optimization problem
    prob = LpProblem("MALP_II_Implementation", LpMaximize)

    # Decision variables
    x = LpVariable.dicts("x", J, 0, 1, LpBinary)  # ambulance placement
    z = LpVariable.dicts("z", [(i, j) for i in J for j in I if i in N[j]], 0, 1, LpBinary)  # coverage assi

    # Objective function: Maximize expected coverage
    prob += lpSum(a[j] * (1 - lpDot([q_i[i] for i in N[j]], [z[i, j] for i in N[j]])) for j in I)

    # Coverage probability constraints (linearized using log transformation)
    for j in I:
        prob += lpSum(z[i, j] * math.log(q_i[i]) for i in N[j]) <= math.log(1 - alpha)

    # Each demand node must have at least one covering ambulance
    for j in I:
        prob += lpSum(z[i, j] for i in N[j]) >= 1

    # Ambulance fleet size constraint
    prob += lpSum(x[i] for i in J) == B

    # Assignment consistency constraints
    for i in J:
        for j in I:
            if i in N[j]:
                prob += z[i, j] <= x[i]

    # Solve
    prob.solve()

    # Print results
    print("MALP II Results:")
    print(f"Status: {LpStatus[prob.status]}")
    print(f"Objective Value: {value(prob.objective)}\n")

    print("Ambulance Locations:")
    placed = [i for i in J if value(x[i]) > 0.5]
```

```python
    for loc in placed:
        print(f"  - Location {loc} (Busy probability: {q_i[loc] * 100}%)")

    print("\nCoverage Details:")
    for j in I:
        covering = [i for i in N[j] if value(z[i, j]) > 0.5]
        prob_coverage = 1 - math.prod([q_i[i] for i in covering])
        print(f"Node {j}:")
        print(f"  Covered by: {covering}")
        print(f"  Coverage probability: {prob_coverage:.2%}")
        print(f"  Meets alpha ({alpha * 100}%): {'Yes' if prob_coverage >= alpha else 'No'}")


# Execute the solver
solve_malp_ii()
```

Appendix 8 - Example code PROFLEET

The following python code was used to solve the PROFLEET example.

```python
from pulp import *
import math

# Sets and parameters
I = [1,2,3,4,5,6]
J = ['A','B','C','D']
a = {1:90, 2:60, 3:120, 4:140, 5:110, 6:80}
alpha = 0.7
p_primary = 3
p_special = 2
p_stations = 3
q_primary = 0.3
q_special = 0.4

# Coverage sets for primary and specialized assets
Np = {
    1: ['A','B'],
    2: ['A'],
    3: ['B','C'],
    4: ['C','D'],
    5: ['D'],
    6: ['C','D']
}
Ns = {
    1: ['B'],
    2: ['A'],
    3: ['C'],
    4: ['C','D'],
    5: ['D'],
    6: ['C']
}

# Compute required units b_i for probabilistic coverage
def req_units(q, alpha):
    return math.ceil(math.log(1-alpha)/math.log(q))

b_primary = {i: req_units(q_primary, alpha) for i in I}
b_special = {i: req_units(q_special, alpha) for i in I}

def solve_profleet():
    prob = LpProblem("PROFLEET_Example", LpMaximize)

    # Decision variables
    x_p = LpVariable.dicts("x_primary", J, 0, 1, LpBinary)
    x_s = LpVariable.dicts("x_special", J, 0, 1, LpBinary)
    z = LpVariable.dicts("z", J, 0, 1, LpBinary)
    y = LpVariable.dicts("y", I, 0, 1, LpBinary)

    # Objective
    prob += lpSum(a[i]*y[i] for i in I)

    # Coverage constraints (linearized)
    for i in I:
        prob += lpSum(x_p[j] for j in Np[i]) >= b_primary[i]*y[i]
        prob += lpSum(x_s[j] for j in Ns[i]) >= b_special[i]*y[i]

    # Resource constraints
    prob += lpSum(x_p[j] for j in J) <= p_primary
    prob += lpSum(x_s[j] for j in J) <= p_special
    prob += lpSum(z[j] for j in J) <= p_stations
```

```
    for j in J:
        prob += x_p[j] <= z[j]
        prob += x_s[j] <= z[j]

    # Solve
    prob.solve()

    print("PROFLEET Results:")
    print("Status:", LpStatus[prob.status])
    print("Max covered demand:", value(prob.objective))
    print("\nStation locations (z):", [j for j in J if value(z[j]) > 0.5])
    print("Primary equipment (x_p):", [j for j in J if value(x_p[j]) > 0.5])
    print("Special equipment (x_s):", [j for j in J if value(x_s[j]) > 0.5])
    print("\nCovered demand points:")
    for i in I:
        print(f"Demand point {i}: {'Covered' if value(y[i]) > 0.5 else 'Not Covered'}")

# Execute the solver
solve_profleet()
```

Appendix 9 - Example code HQM

The following python code was used to solve the HQM example.

```
from pulp import *
import math

# Parameters
I = range(1, 7)  # 6 demand nodes
J = range(1, 4)  # 3 candidate locations
B = 2# ambulances to deploy
a = {1: 100, 2: 150, 3: 120, 4: 130, 5: 110, 6: 140}  # demand node populations
q = 0.3  # coverage probability
alpha = 0.9  # desired coverage probability

# Coverage sets (station to nodes)
N = {
    1: [1, 2, 5],  # S1 covers A, B, E
    2: [1, 3, 4],  # S2 covers A, C, D
    3: [2, 4, 5, 6]  # S3 covers B, D, E, F
}


def solve_hqm():
    prob = LpProblem("HQM_Optimization", LpMaximize)

    # Binary variables: x[j] = 1 if station j is selected, else 0
    x = LpVariable.dicts("x", J, 0, 1, LpBinary)

    # Binary variables: y[i] = 1 if demand node i is covered, else 0
    y = LpVariable.dicts("y", I, 0, 1, LpBinary)

    # Objective function: maximize covered population
    prob += lpSum(a[i] * y[i] for i in I)

    # Calculate required number of servers (m)
    m = math.ceil(math.log(1 - alpha) / math.log(q))

    # Constraints
    for i in I:
        # Find stations that cover demand node i
        covering_servers = [j for j in J if i in N[j]]
        # Ensure that if a node i is covered, the selected stations' sum is >= m * y[i]
        prob += lpSum(x[j] for j in covering_servers) >= m * y[i]

    # Exact number of ambulances to deploy
    prob += lpSum(x[j] for j in J) == B

    # Solve the LP
    prob.solve()

    # Output results
    print(f"Status: {LpStatus[prob.status]}")
    print(f"Covered Population: {value(prob.objective)}")

    print("Ambulance Locations:")
    for j in J:
        if value(x[j]) == 1:
            print(f"S{j}")

# Execute the solver
solve_hqm()
```